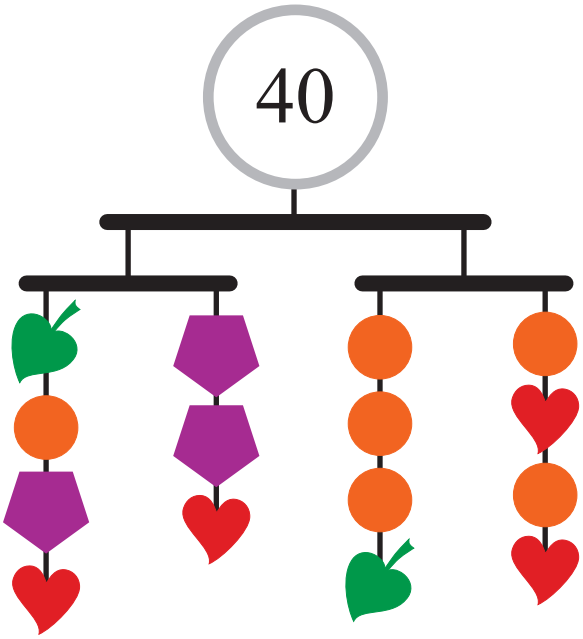
Transition to Algebra



Field Test Materials

Book 1

EDC Learning transforms lives.

The Transition to Algebra materials are being developed at Education Development Center, Inc. in Waltham, MA. More information on this research and development project is available at ttalgebra.edc.org.

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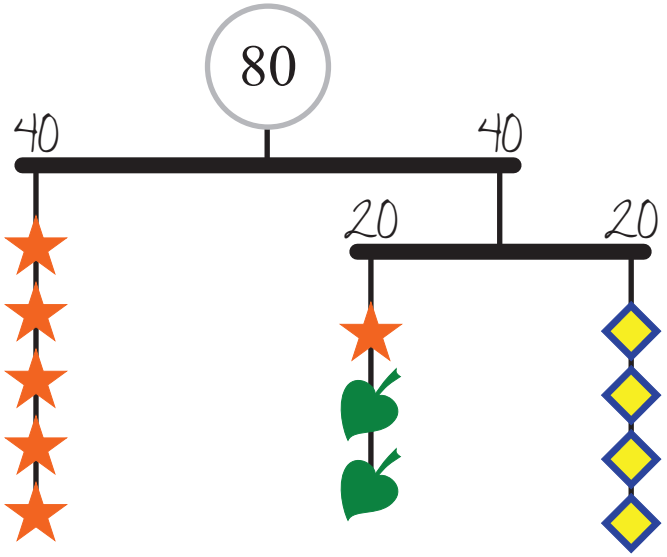
Transition to Algebra
Book 1 Contents:

Unit 1: Language of Algebra



$5 \bigstar = 40$

$\bigstar = 8$

$\bigstar + 2 \text{🍃} = 20$

$8 + 2 \text{🍃} = 20$

$2 \text{🍃} = 12$

$\text{🍃} = 6$

$4 \diamond = 20$

$\diamond = 5$

Transition to Algebra

Unit 1: Language of Algebra

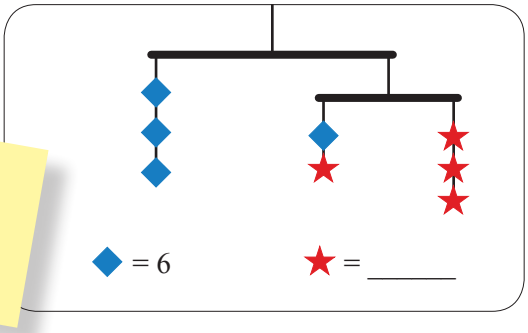
What is Algebra?

Algebra is two things: a convenient language for expressing patterns and relationships you know, and a logical system for figuring out things that you don't know yet. In this unit, you will learn to describe mathematical relationships and patterns with pictures, English, and algebraic symbols. You will also learn to use your own logical thinking to work out the values of numbers you don't know. Puzzles will be an important part of this work because puzzles are perfect examples of many mathematical ways of thinking. The goal is for you to take the logical thinking you already have and learn how to express that thinking in the language of algebra.

Lessons in this Unit:

1: Exploring Number Magic Tricks
2: Creating Number Magic Tricks
3: Balancing Mobile Puzzles
4: Describing Tricks and Puzzles
5: What Else Can We Do?

You will learn to solve puzzles like these...



Words	Example	Pictures	Descriptions of Pictures	Abbreviation
Think of a number.	6		a bucket	b
Multiply by 2.	12		2 buckets	
Add 4.			2 buckets and 4	2b + 4
Divide by 2.	8			b + 2
Subtract your original number.	2		2	2

Algebraic Habits of Mind: Puzzling

The problems we encounter in the "real world" (our family life, work, personal issues, etc.) don't ask us what chapter we've just studied. They usually don't tell us what we need to do. They don't always tell us what question to answer. And they almost never tell us where to begin. They just happen. To succeed, we must figure out what we know, what question to ask, what else we might need to know, and where to start.

Use these materials to learn math through puzzles and logic problems that take the same approach.

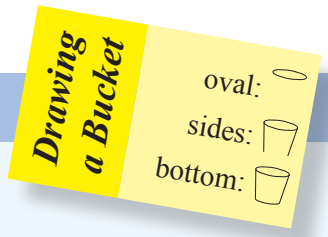
Learn to ask yourself:

- What questions can I ask about this situation or information?
- How can I change this problem into an easier problem? Will that help me with this harder one?
- What clues do I have? What other information do I need?
- How can I find that information from what I already know?

1-1 Exploring Number Magic Tricks

①

Words	Pictures	Ian	Maria	Jacob
Think of a number.		4	0	
Add 5.		9		
Multiply by 2.				14
Subtract 2.				12
Divide by 2.				
Subtract your original number.				



Thinking out Loud

Michael, Lena, and Jay are working on problem #1.

Michael: How can we subtract Jacob's original number?! We don't know his original number!

Lena: Well, the table says that Jacob got 14 when the picture was  (2 buckets and 10).

So 2 buckets plus 10 is 14. *(Lena draws*  = 14 *on the board.)*

I can "see" 10 of Jacob's 14, so the remaining 4 must be split between those two buckets.

(Lena removes 10 *from each side:*  = 4.*)*

Michael: Oh! So  = 4 and Jacob's original number must have been 2!

Pausing to Think If  = 14, how do they know that  = 4?

Jay: I found Jacob's number differently! I went backwards. He got 14 after multiplying by 2, so...

Michael: He must have been thinking of the number 7!

Jay: Yeah, so I wrote that. And then he got that 7 by adding 5 to his original number.

Michael: So, either way he started with 2!

② If Carla got 22 when the picture was  , what was her original number ()? _____

③ If  = 16, what is  ? _____

④ If  = 18, what is  ? _____

Fill in the empty boxes in this Think-of-a-Number trick table.

⑤

Words	Pictures	Kayla	Raj	Ben
Think of a number.			7	
Multiply by 2.		18		
Add 4.				64
Divide by 2.				32
Subtract your original number.				

⑥ If  = 16, what is  ? _____

⑦ If  = 18, what is  ? _____

Some parts of this trick are missing. You have enough clues to figure them out. Fill in all the missing parts.

⑧

Words	Pictures	Jing	Ben	Ian
Think of a number.				
Multiply by 6.		18		
Add 8.		26	50	68
		13	25	
				30
Divide by 3.				10

⑨ If  = 34, then  = _____.

⑩ If  = 50, then  = _____.

Find the pattern in the table below. Draw what the trickster was picturing and find the results and starting numbers.

⑪

	7	15	101	39	99		0	-1		
	9	17	103			3		1	0	$3\frac{1}{2}$

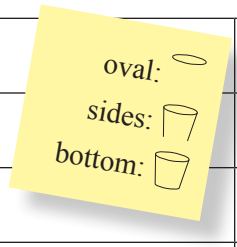
⑫ If you were playing this trick on someone, what instruction would you give after "Think of a number?"

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

Ⓐ

Words	Pictures	Imani	Raj	Eva
Think of a number.		2		
Add 2.		4	12	
Multiply by 3.			36	
Subtract your original number.				20
			13	10
Subtract your original number.			3	



Ⓑ If Malika also got 10 when the picture was  , what was her original number ()? _____

Ⓒ If  = 27, what is  ? _____

Ⓓ If  = 21, what is  ? _____

Ⓔ

Words	Pictures	Mali	Asher	Luis
Think of a number.		8	3	
Add 6.		14	9	21
				42
		26		40
		13		20
Subtract your original number.			5	

Ⓕ If Brandon got 15 when the picture was  , what was his original number ()? _____

Find the pattern in the table below. Draw what the trickster was picturing and find the results and starting numbers.

Ⓖ

	1	7	6	3	13	93			-1	0.5
	8	14	13				77	7		

Ⓗ If you were playing this trick on someone, what instruction would you give after "Think of a number?"

1-2 Creating Number Magic Tricks

Create your own trick that **always results in the number 9**. Have two other students try it and write their answers here.

①

Words	Pictures	_____	_____
Think...			

You don't have to use all the rows.

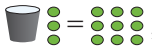
You don't have to use all the rows.

② Maria thought of the number 2. After one step, she got 8. Name two ways she could have gotten to 8.

③ Hiroshi thought of the number 5. After *two* steps, he had 11. Name ways that could have happened.

Algebraic Habits of Mind: Communicating Clearly

Communicating your ideas clearly - in writing, speech, or mathematics - not only helps you get more credit for your work, but also helps you learn. When you communicate clearly, other people understand you better and can offer better suggestions to improve your thinking and strategies.

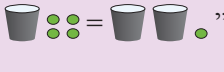
④ If  , then  = _____.

⑤ If  = 25, then  = _____.

⑥ If  + 11 = 35, then  = _____.

⑦ If  + 1 =  + 3, then  = _____.

Discussing Together

What number can  represent so that the statement "  " will be true?

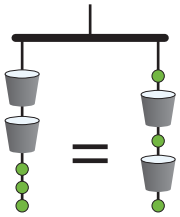
Discussing Together

How many different numbers can the ● represent in the statement ●●● = ●●●?

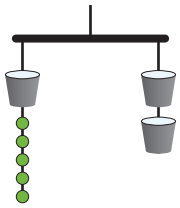
How many different numbers can the ● represent in the statement ●●● = ●●?

In each of these problems a dot (●) = 1.

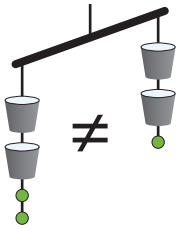
⑧ This mobile *always balances*. Why?



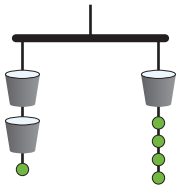
⑨ This mobile *only balances when* the buckets represent a certain number. What number makes it balance?



⑩ This mobile *never balances* no matter what number the bucket represents. Why?

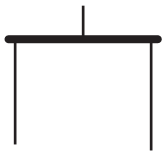


⑪ Does this mobile balance *sometimes, always, or never?*



If sometimes, *when?*

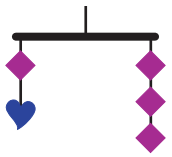
⑫ Draw a mobile that *always* balances.



⑬ Create an equation or mobile that can *never* balance.

⑭ Draw something that *only balances when* ● = 3.

⑮ How many ◆ in a ♥? _____.



Find the pattern in the table below. Draw the missing step and find the other numbers.

⑯

	7	12		9	14	14.5				0
● – 3	4	9	25				30	29	0	

⑰ If you were playing this trick on someone, what instruction would you give after "Think of a number?"

Additional Practice Problems

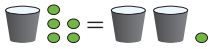
Select problems that will help you learn. Do some problems now. Do some later.

(A) Create your own trick that **always results in the number 4**. Have two people do the trick with different numbers.

Words	Pictures	_____	_____
Think...			

You don't have to use all the rows.

You don't have to use all the rows.

(B) If  then  = _____.

(C) If  then  = _____.

(D) If  = 14, then  = _____.

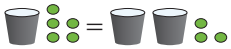
(E) If  = 15, then  = _____.

(F) Kayla is thinking of a number. If she doubles her number, the answer is 22. What was her original number? _____

(G) If  + 4 = 50, then  = _____.

(H) If  – 5 = 6, then  = _____.

What minus 5 equals 6?

(I) What number can  represent so that the statement "  " will be true? _____

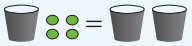
Find the pattern in the table below. Draw the missing step and find the other numbers.

(J)

	11	8.5	19	14	24		8		6	
	4	1.5	12			27		0		-7

(K) If you were playing this trick on someone, what instruction would you give after "Think of a number?"

Thinking out Loud

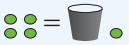
What number can  represent so that the statement "  " will be true?

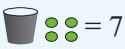
Lena: I never thought about it before, but that statement uses an equals sign even though the two sides don't look the same!

Jay: They aren't the same, but they do have the same value.

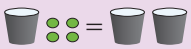
Michael: That's why it says "equals." So the questions is, *when* do they have the same value? When will it be true?

Lena: Let's see what has to be in a bucket for the two amounts to be the same. To find out what one bucket represents, we can remove the matching stuff on both sides until only one bucket is left.

Jay: So, we can remove the matching buckets. *(Jay crosses out two buckets:*  *)* That leaves us with  and we remove the matching ones. *(Lena draws*  *)*

Michael: So  = 3. That makes sense! If  = 3, then  = 7 and  = 7. So they're equal!

Discussing Together

If  = 2, would the statement "  " be true? Why or why not?

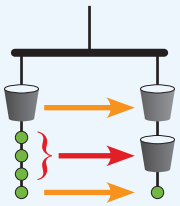
Thinking out Loud

What number can  represent so that the statement "  " will be true?

Jay: When I did it, I started in a completely different way! I knew that 1 bucket plus 4 is the same number as 2 buckets plus 1. So, I pictured them balancing on a mobile.

Michael: Oh, and because they're equal, the two sides have to balance!

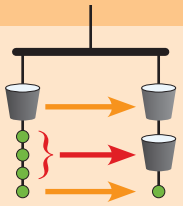
Jay: Yeah! We can imagine the buckets and ones hanging from the strings. Just like before, the bucket holds my original number, but we can't see inside. Anyway, I saw that the top of each side is a bucket and the bottoms both have ones, so the middles have to match up too!



Michael: I get it! To make it balance, that bucket on the right has to weigh the same as the 3 ones on the left. That's how the chunks of stuff match up!

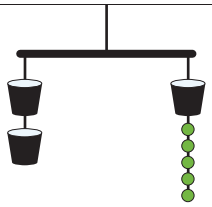
Algebraic Habits of Mind: Using Structure

Jay looks at the mobile and sees that a whole chunk of the left side matches with one piece on the right side. This way of thinking can often make problems simpler to solve.



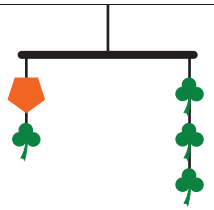
Every beam in these mobiles is balanced. Find the missing weights.

1



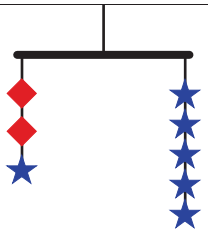
● = 1 ▲ = _____

2



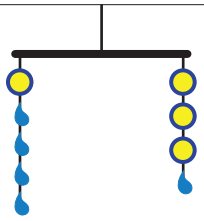
⬟ = _____ ♣ = 2

3



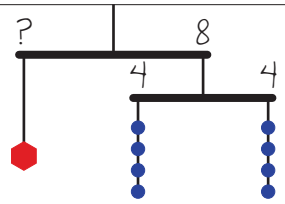
◆ = _____ ★ = 3

4



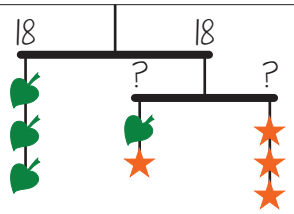
💧 = 4 ○ = _____

5



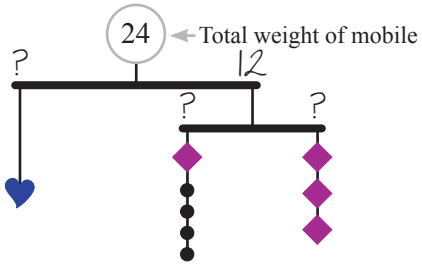
● = 1 ⬢ = _____

6



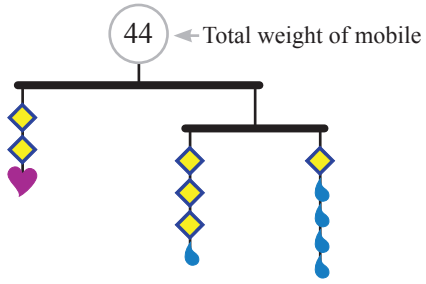
🍃 = 6 ★ = _____

7



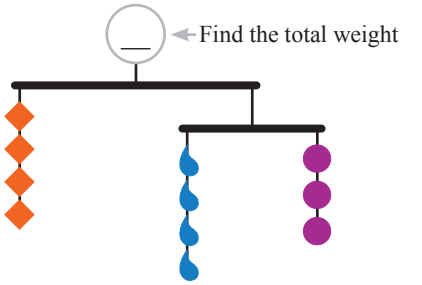
● = 1 ◆ = _____ 💜 = _____

8



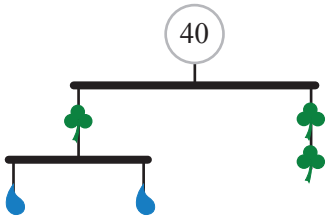
💜 = _____ ◇ = _____ 💧 = 2

9



◆ = 6 💧 = _____ ● = _____

10

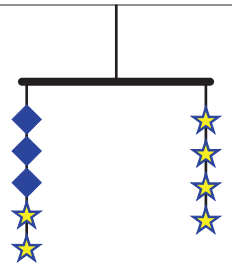


♣ = _____ 💧 = _____

Additional Practice Problems

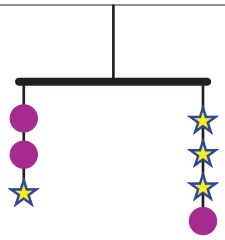
Select problems that will help you learn. Do some problems now. Do some later.

A



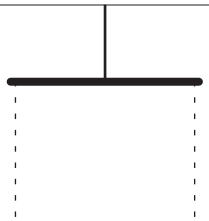
= 6 = _____

B



= 6 = _____

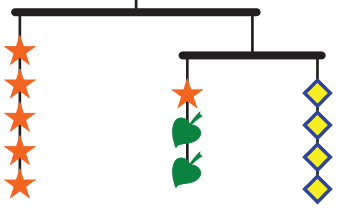
C



Create your own mobile puzzle.

D

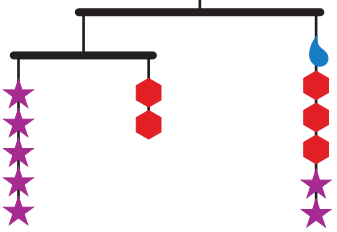
80 ← Total weight of mobile



= _____ = _____ = _____

E

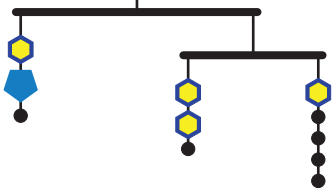
120 ← Total weight of mobile



= 15 = _____ = _____

F

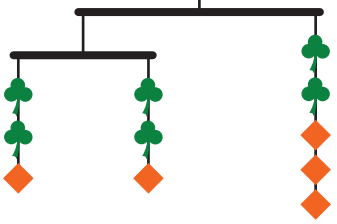
28



= 1 = _____ = _____

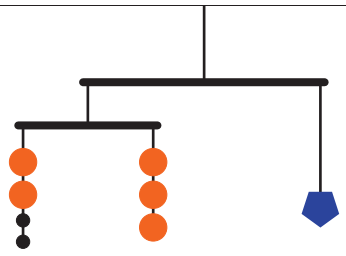
G

64



= 8 = _____

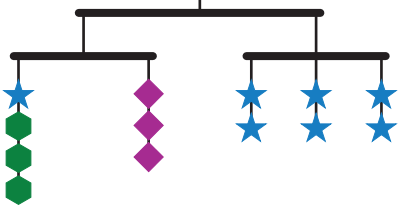
H



= 1 = _____ = _____

I

72



= _____ = _____ = _____

1-4 Describing Tricks and Puzzles

Remember this trick? Draw and describe the pictures, and abbreviate your descriptions.

①

Words	Pictures	Description of Pictures	Abbreviation
Think of a number.		a bucket	b
Add 5.		a bucket and 5	$b + 5$
Multiply by 2.			$2b + 10$
Subtract 2.			
Divide by 2.			
Subtract your original number.			

Discussing Together

Jessica is thinking of a number. She adds something to it. **The sum is the number she just added.** What was her starting number? Why?

Try a few numbers to test your ideas.

Luis is thinking of a number. He multiplies it by something. **The product is the number he just multiplied by.** What was his starting number? Why?

② What was Jessica's starting number? _____
Show 3 examples of adding something to her number.

③ What was Luis's starting number? _____
Show 3 examples of multiplying it by something.

Some parts of this trick are missing. You have enough clues to figure them out. Fill in all the missing parts.

④

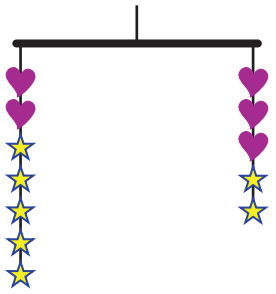
Words	Pictures	Description of Pictures	Abbreviation
Think of a number.		a bucket	
			$b + 2$
Multiply by 3.			
			$2b + 6$
Divide by 2.			
			3

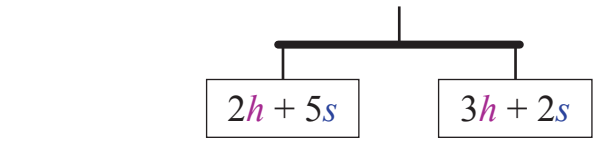
Find the pattern to complete the table below.

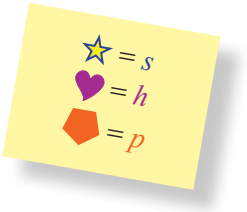
⑤

	4	7	9		100		0	$\frac{1}{2}$	
		14		20		2000			2b

⑥ These two mobiles mean the same thing. Solve them both.

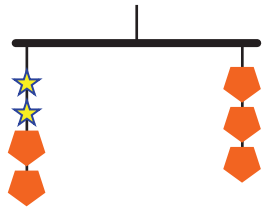


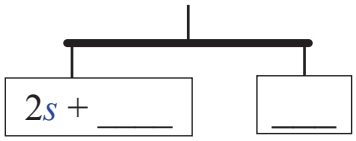




☆ = 1 ♥ = _____ $s = 1$ $h =$ _____

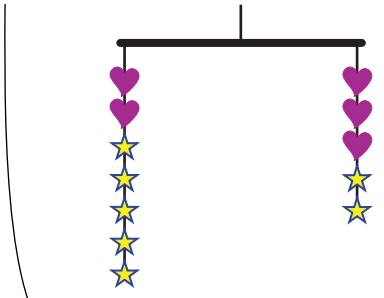
⑦ These two mobiles also mean the same thing. Complete the second one, and solve them both.





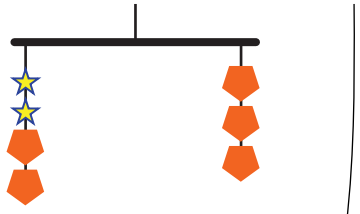
☆ = 1 ⬟ = _____ $s = 1$ $p =$ _____

⑧ This equation describes the first mobile.



$2h + 5s = 3h + 2s$

Write an equation that describes the second mobile.



_____

Translate these two equations into algebraic notation. Use c for ♣ and d for 💧.

⑨ ♣♣ + 4 = ♣ + 11

2c + 4 = _____

⑩ 💧💧💧💧 + 3 = 💧💧💧 + 8

_____ = _____

⑪ Represent this equation using pictures:

$2b + 5 = b + 9$

⑫ What is the value of d in the equation above? _____

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

A Some parts of this trick are missing. You have enough clues to figure them out. Fill in all the missing parts.

Words	Pictures	Description of Pictures	Abbreviation
Think of a number.		a bucket	b
		a bucket and 4	
			$2b + 8$
Subtract 6.			
			$b + 1$

We can use n for 'number.'

B

4	7	0.5	3			0		10	100	n
20	35	2.5		55	40		5			5n

C

6	4	11	5	8				3		n
42	28	77			0	49	7		63	

Translate these two equations into algebraic notation. Use d for ◆ and x for ⬢.

D ◆◆◆ – 1 = ◆◆ + 4

3d - 1 = _____

E ⬢⬢⬢⬢ – 3 = ⬢⬢⬢ + 8

_____ = _____

F Represent this equation using pictures:

$4b + 3 = 2b + 11$

G If  + 7 = 20, then  = _____.

H If $b + 7 = 20$, then b = _____.

You can draw anything for b or just use a bucket: 

1-5 What Else Can We Do?

① Do these two number tricks give the same result? Draw pictures or write abbreviations if you like.

Trick #1	Trick #2
Think of a number.	Think of a number.
Add 4.	Multiply by 2.
Multiply by 2.	Add 4.
Subtract 2.	Subtract 2.
Divide by 2.	Divide by 2.
Subtract your original number.	Subtract your original number.

So, do they give the same result? _____ Why?

Arrange the steps below to create a trick that always results in the number you started with. Test your trick with four starting numbers, and write abbreviations for the pictures you would draw.

Add 12.	Divide by 2.	Multiply by 2.	Subtract 6.

②

Words	Test 1	Test 2	Test 3	Test 4	Abbreviation
Think of a number.					b
					b

Discussing Together

How does this trick work? How can you be sure that the trick *always* results in the original number?

n stands for 'number.'

③

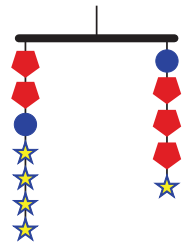
3	11	5	202		1				10	*n*
6	22	10	404	30			0			
7	23	11		31		101		13		$2n + 1$

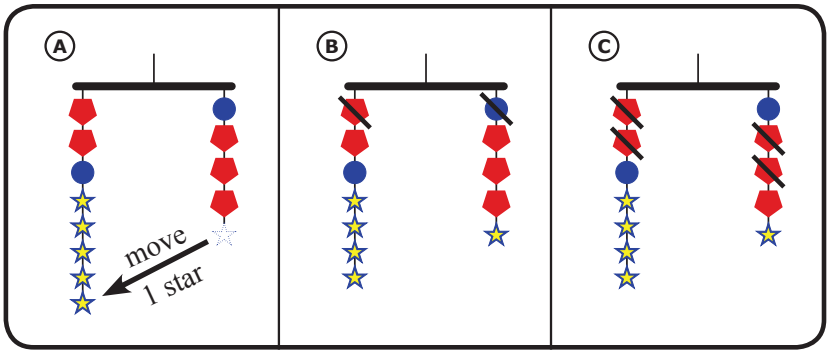
④ Imani is describing her work on this problem, but parts of her answer are missing. Can you figure out what she did?

What number can 🪣 represent so that the statement "🪣🪣🪣●●●=🪣🪣●●●●●" is true?

First, I rewrote the problem in algebra like this: $3b + 5 = 2b +$ _____.
Then, I imagined crossing out ___ buckets on each side so one side had none.
So I wrote: $b + 5 =$ _____.
Next, I imagined crossing out _____.
Finally, I could see that the bucket always has to be ___.

⑤ Which of the following is a proper move that will always keep the mobile balanced?





Explain how you know.

Solve these equations.

⑥ If 🪣🪣 + 1 = 🪣 + 6, then 🪣 = _____.

⑦ If $2b + 1 = b + 6$, then $b =$ _____.

⑧ If $3b = 33$, then $b =$ _____.

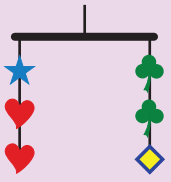
⑨ If 🪣🪣 + 2 = 🪣 + 5, then 🪣 = _____.

⑩ If $2b + 2 = b + 5$, then $b =$ _____.

Discussing Together

Brandon has created a puzzle for the class. He says that both of these first two mobiles balance:

Then he asks if this third mobile balances too.



Does it? Why or why not?

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

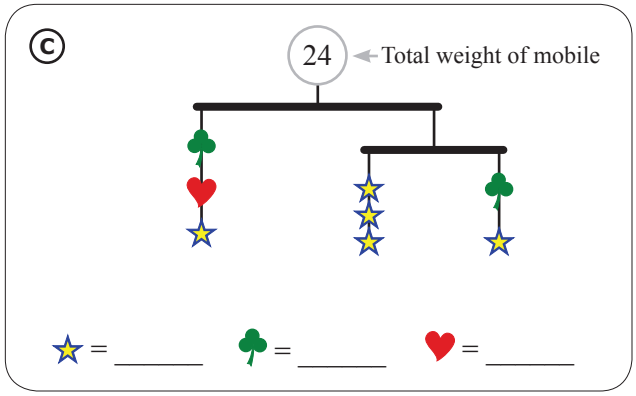
Find the pattern in the table below. Draw what the trickster was picturing and find the results and starting numbers.

Ⓐ

2	4	8		6				7		n
14	28	56	21		63	0	35		7	

Ⓑ If you were playing the trick above on someone, what instruction would you give after "Think of a number?"

Ⓒ



Solve these equations.

Ⓓ If 🪣🪣 + 7 = 🪣 + 10, then 🪣 = _____.

Ⓔ If $2b + 7 = b + 10$, then $b =$ _____.

Ⓕ If 🪣🪣🪣 + 1 = 🪣 + 11, then 🪣 = _____.

Ⓖ If $3b + 1 = b + 11$, then $b =$ _____.

Ⓗ

9	101	7	3				-1	🪣	n
11	103	9		25	3		1	🪣•	
33	309	27				6			$3n + 6$

Find all of the missing parts of this Think-of-a-Number trick.

Ⓘ

Words	Pictures	Eva	Abbreviation
Think of a number.	🪣		
	🪣🪣🪣		
	🪣🪣🪣🪣		
Add 8.		32	$4b + 8$
		8	$b + 2$
Subtract your original number.			

Unit Additional Practice Problems

This page looks like the unit exam.
Use it to help you prepare.

Some parts of this trick are missing.
You have enough clues to figure them out. Fill in all the missing parts.

①

Words	Pictures	Eva	Abbreviation
Think of a number.			b
			
Multiply by 2.			
		20	$2b + 6$
			$b + 3$
Subtract your original number.		3	

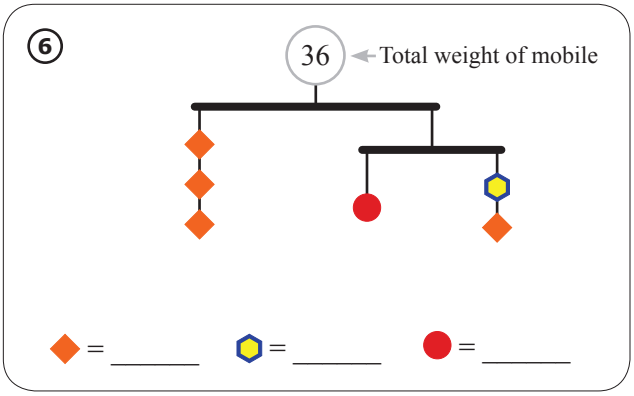
② If  = 49, then  = _____.

③ If ● + 25 = ● ● + 15, then ● = _____.

④ If  = 21, then  = _____.

⑤ If $b - 6 = 5$, then b = _____.

⑥



⑦

6	15	18	21	47	3				n
9	18	21	24			36			
18	36	42	48				60		$2n + 6$

Represent this equation using pictures.

⑧ $4b + 5 = 3b + 7$

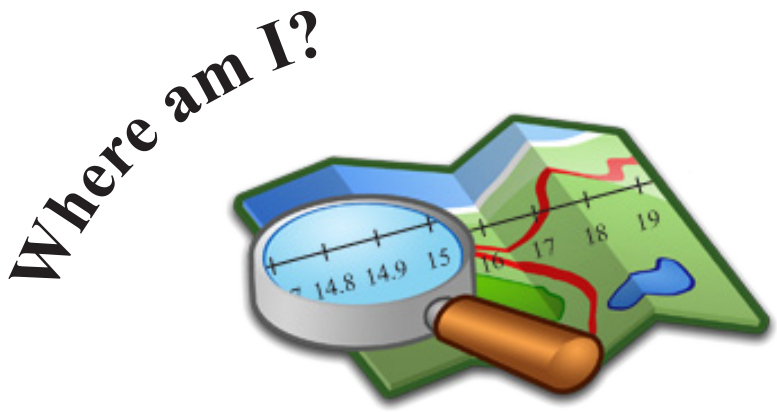
Translate this equation into algebra. Let ★ = s.

⑨ ★ ★ – 8 = ★ + 6

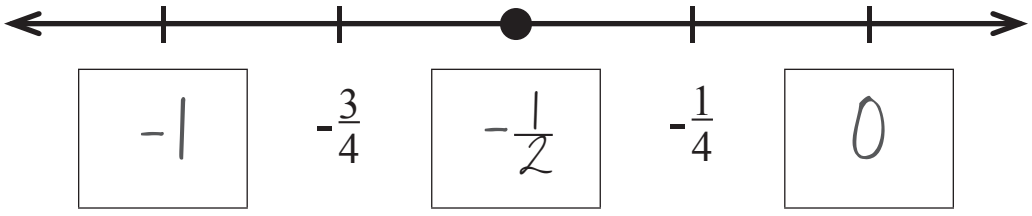
$2s$ - _____ = _____

⑩ Mali is thinking of a number. If she multiplies her number by 7, the answer is 42.
What was his original number?

Unit 2: Geography of the Number Line



I am halfway between $-\frac{3}{4}$ and $-\frac{1}{4}$. Where am I? $-\frac{1}{2}$



Transition to Algebra

DRAFT

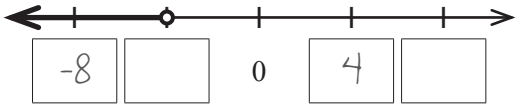
Unit 2: Geography of the Number Line

Lessons in this Unit:

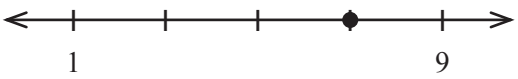
1: Where Do Negatives Live?
2: Numbers between Numbers
3: Where Am I? Number Puzzles
4: Zooming in Differently
5: Subdividing the Number Line
6: Ordering Logically
7: Revenge of the Where Am I? Number Puzzles

You will learn to solve puzzles like these...

I am less than -4. Where am I? $x < -4$

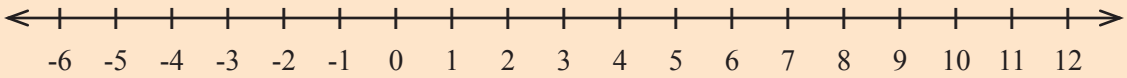


I am $\frac{3}{4}$ of the way from 1 to 9. Where am I?

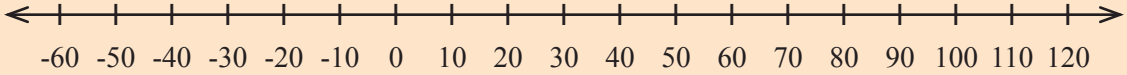


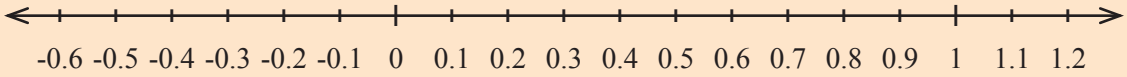
Algebraic Habits of Mind: Using Structure

In school so far, you may have been taught "rules" to use as you solve arithmetic problems. In algebra, you will learn more about how numbers and operations are organized so that you can use your common sense more and not depend on so many rules. One important structure is the number line which is how numbers are organized. Mathematicians picture numbers as "addresses" on a line, organized like this:



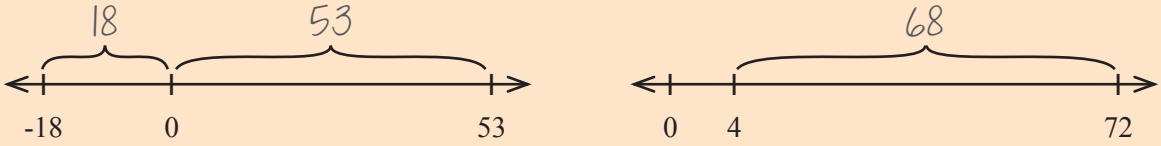
This kind of image lets you figure out easily how far -6 is from 10 and see that 2 is exactly halfway in between. It also lets you see how the structure of the number line is really the same if we zoom in or out with decimals:





You can also see that it's even the same kind of structure when we break up the number line with fractions.:

The number line can also help with operations. By looking at *where* numbers are, you can see *why* you need to use addition to figure out how far -18 is from 53, but subtraction to figure out how far 4 is from 72.



Thinking about the structure of the number line is a very useful tool in algebra.

 DRAFT

2-1 Where Do Negatives Live?

① What is the next number in this pattern? 11, 8, 5, 2, _____

Try imagining a number line.

Thinking out Loud

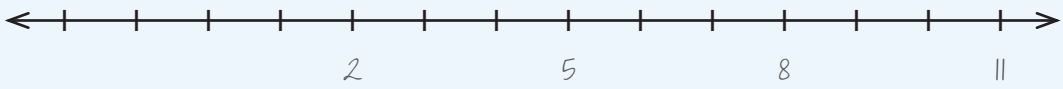
Michael , Lena, and Jay are working on problem #1.

Michael: ! Is it a negative or a fraction? Maybe it's a decimal. Something's got to change...

Lena: Yeah, something's going to change. The numbers go down by 3, so it will be 2 minus 3?

Pausing to Think What does Lena mean by "the numbers go down by 3?"

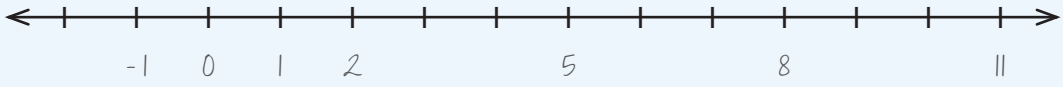
Jay: I pictured it on a number line. *(Jay draws a number line and marks 11, 8, 5, and then 2.)*



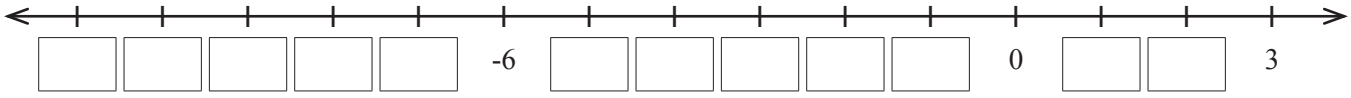
Then I counted down 3 more and got to...

Michael: Oh, I know! 1, 0, -1. You got to -1. We subtracted three so many times we went below zero!

(Michael fills in a few more numbers on the number line to show how he got to -1.)



② Finish labeling this number line to help you answer the problems below.



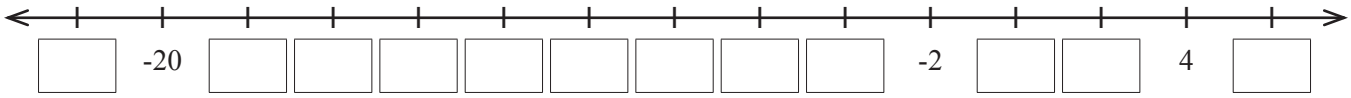
③ What are the next three numbers in this pattern? 9, 7, 5, 3, _____ , _____ , _____

④ What is the next number in this pattern? 10, 6, 2, _____

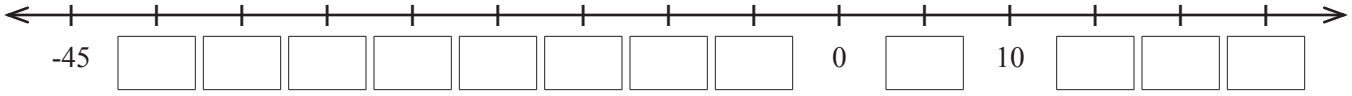
⑤ What is the *distance* between -2 and 3? _____

Some number lines go up by 1 each time; some go up by 2 or something else, but all number lines go up by their own same amount each time. Like rulers, they each have their own *scale*.

⑥ This number line has a different scale. Finish labeling it.



⑦ This number line has yet another scale. Finish labeling it to help you answer the problems below.

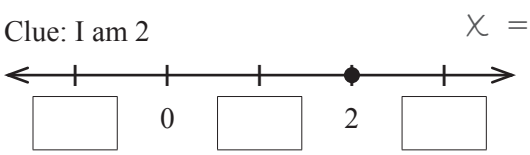


⑧ What are the missing numbers in this pattern? 15, 5, _____ , -15, _____ , _____ , -45

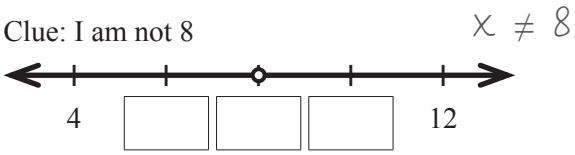
⑨ What are the missing numbers in this pattern? 30, _____ , 0, _____ , _____ , -45

Where am I? Label these number lines using the clues. Keep an eye out for different scales!

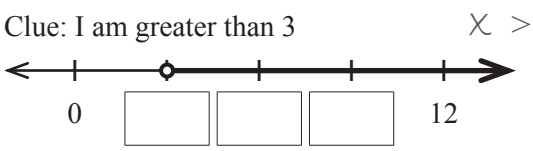
⑩ Clue: I am 2



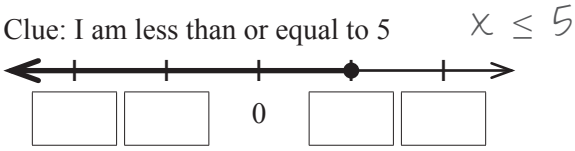
⑪ Clue: I am not 8



⑫ Clue: I am greater than 3



⑬ Clue: I am less than or equal to 5

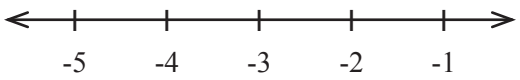


Dark lines and **filled** circles (●) mark numbers that make the statement **true**. They are **solutions**.

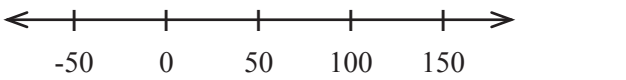
Light lines and **empty** circles (○) mark numbers that make the statement **false**. They are ***not*** **solutions**.

Where am I? Mark all possible solutions on each number line and complete the algebraic expression that describes them.

⑭ Clue: I am not -4



⑮ Clue: I am less than 100

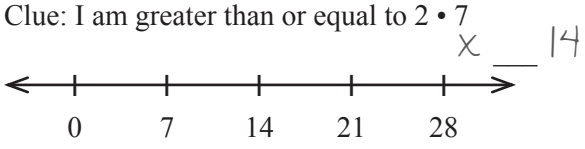


Discussing Together

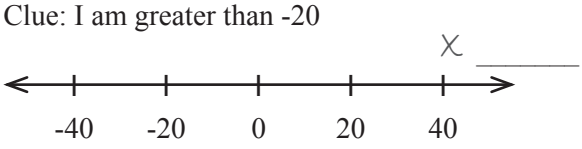
How did you decide what kind of circle to draw: empty or filled?

How did you decide which symbols to write: =, ≠, < , > , ≤ , or ≥?

⑯ Clue: I am greater than or equal to 2 • 7



⑰ Clue: I am greater than -20



You can use the number lines above to answer these questions.

⑱ What is the *distance* between 0 and 21? _____

⑲ What number is *halfway* between 0 and 20? _____

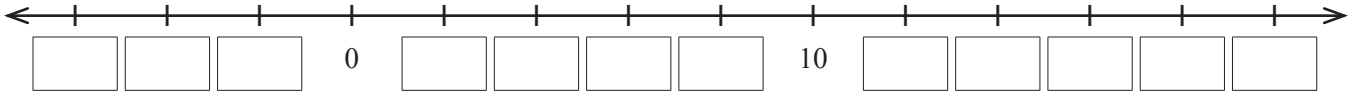
⑳ What is the *distance* between 7 and 28? _____

㉑ What number is *halfway* between -20 and 40? _____

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

A Finish labeling this number line and use it to answer the problems below.



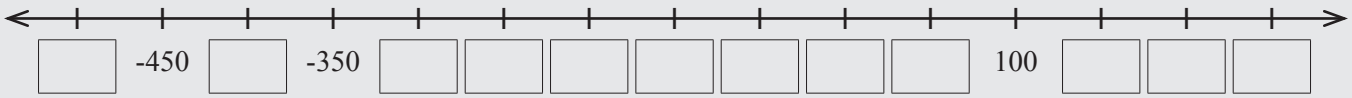
B What are the next three numbers in this pattern? 20, 16, 12, 8, _____ , _____ , _____

C What are the missing numbers in this pattern? 12, 10 , 8, _____ , 4, _____ , _____ , _____ , -4 , _____

D What number is *halfway* between -4 and 10? _____

E What is the *distance* between -6 and 4? _____

F Finish labeling this number line. Notice the different scale.

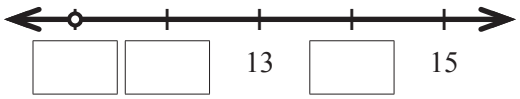


G What number is *halfway* between -350 and -50? _____

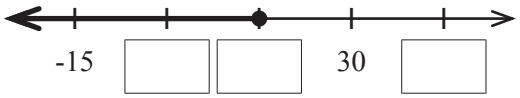
H What is the *distance* between -350 and 150? _____

Where am I? Label these number lines using the clues. Keep an eye out for different scales!

I Clue: I am not 11 $x \neq 11$



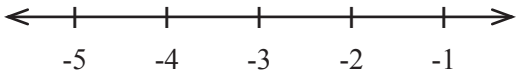
J Clue: I am less than or equal to 15 $x \leq 15$



Numbers with a ● make the statement **true.** Numbers with a o make the statement **false**.

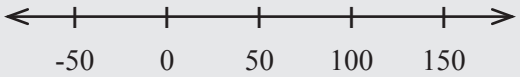
Where am I? Mark all possible solutions on each number line and write an algebraic expression to describe them.

K Clue: I am greater than or equal to -2 x ___ -2



L What is the *distance* between -5 and -1? _____

M Clue: I am less than 50 _____

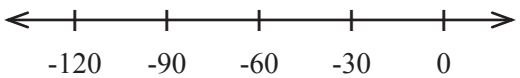


N What number is *halfway* between -50 and 150? _____

O Clue: I am not -12 _____

P What is the *distance* between -18 and 6? _____

Q Clue: I am greater than -30 _____



R What number is *halfway* between -90 and -30? _____

2-2 Numbers between Numbers

Thinking out Loud

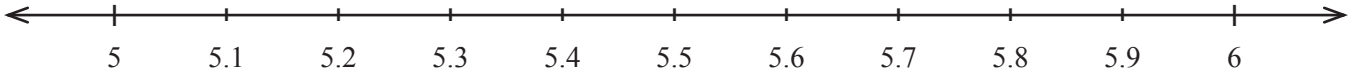
Michael and Lena are trying to determine what number you can multiply by itself to get 29.

Michael: The square of 5 is 25 since 5 • 5 = 25 and the square of 6 is 36. So, the number you multiply by itself to get 29 must be between 5 and 6. How can we check the numbers in between?

Lena: Let's zoom in and look at the numbers between those numbers!

Zooming In to find $\sqrt{29}$:

① Use this zoomed in number line and a calculator to find two adjacent numbers that $\sqrt{29}$ is between.



$5^2 = 25$ ← 29 isn't here. So $\sqrt{29}$ isn't between 5 and 5.1

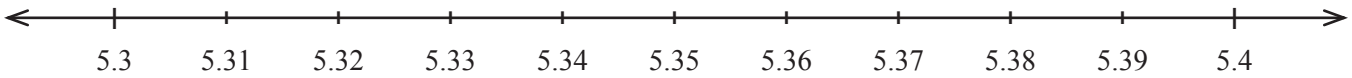
$5.1^2 = 26.01$

② A positive number that you can multiply by itself to get 29 must be between 5 . ___ and 5 . ___ .

So $\sqrt{29}$ starts with 5 . ___ .

Zooming in *More*:

③ Use this zoomed in *more* number line and a calculator to find two adjacent numbers that $\sqrt{29}$ is between.



$5.3^2 = 28.09$ ← 29 isn't here. So $\sqrt{29}$ isn't between 5.3 and 5.31

$5.31^2 = 28.1961$

The goal is to *approximate* (get close to) the *square root* of 29, or just $\sqrt{29}$.

④ A number that you can multiply by itself to get 29 must be between 5 . ___ ___ and 5 . ___ ___ on the number line. So $\sqrt{29}$ starts with 5 . ___ ___ .

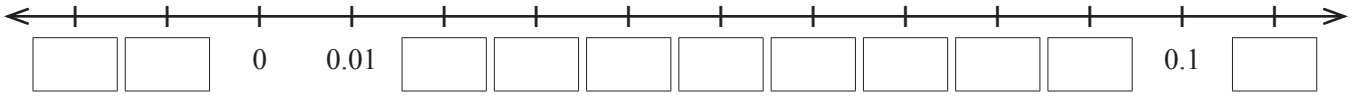
Discussing Together

If you wanted a more *precise* (exact) answer than this approximation, what could you do?

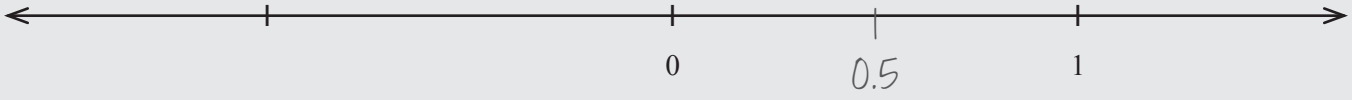
Complete these **Zooming In** number lines.

⑤

⑥



⑦ Place these numbers on the number line below: 0.5 -1 1.1 0.2 -0.8 -1.2 0.7 -0.1 -1.2

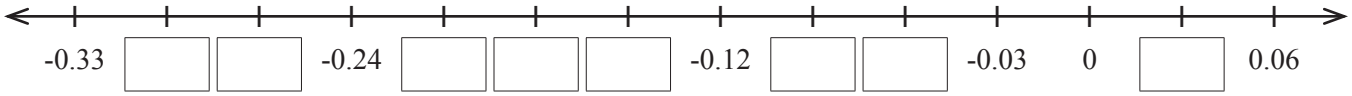


⑧ Which is greater, -0.8 or -1.2 ?

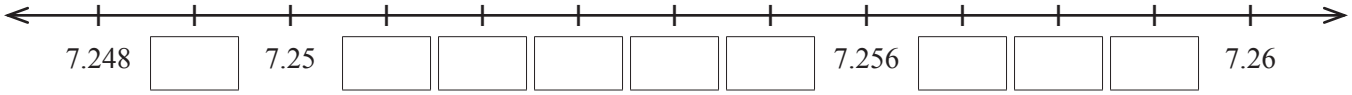
⑨ What number is exactly halfway between 1.1 and -0.1? _____

Complete these **Zooming In** number lines.

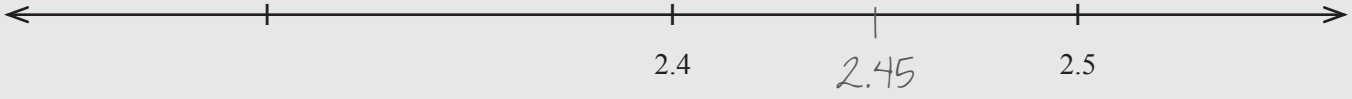
⑩



⑪



⑫ Place these numbers on the number line below: 2.45 2.3 2.41 2.33 2.51 2.55 2.38 2.28

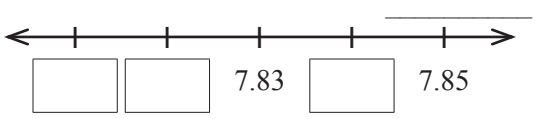


⑬ Which is greater, 2.32 or 2.4? _____

⑭ What number is exactly halfway between 2.28 and 2.38? _____

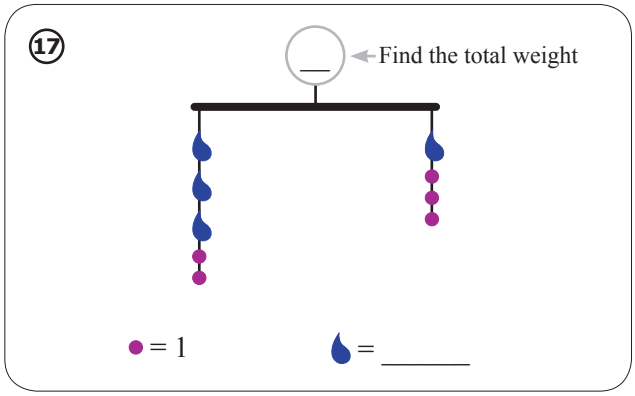
Where am I? Mark all possible solutions on each number line and write an algebraic expression to describe them.

⑮ Clue: I am greater than 7.81



⑯ What is the *distance* between 7.82 and 7.85? _____

⑰



Additional Practice Problems

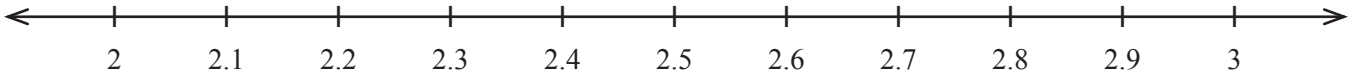
Select problems that will help you learn. Do some problems now. Do some later.

What number can you multiply by itself to get $\sqrt{8}$?

Ⓐ Since 2 • 2 = 4 and 3 • 3 = 9, the positive number that you can multiply by itself to get 8 must be between ____ and ____. So $\sqrt{8}$ starts with ____.

Zooming In to find $\sqrt{8}$:

Ⓑ Use this zoomed in number line and a calculator to find two numbers that $\sqrt{8}$ is between.



$2^2 = 4$

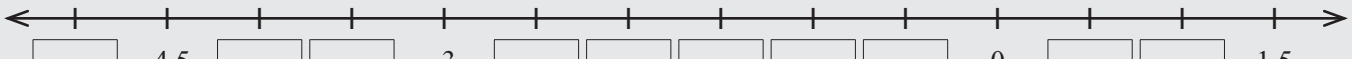
$2.1^2 = 4.41$ ← 8 isn't here. So $\sqrt{8}$ isn't between 2 and 2.1

Ⓒ A positive number that you can multiply by itself to get 8 must be between 2 . ___ and 2 . ___ .

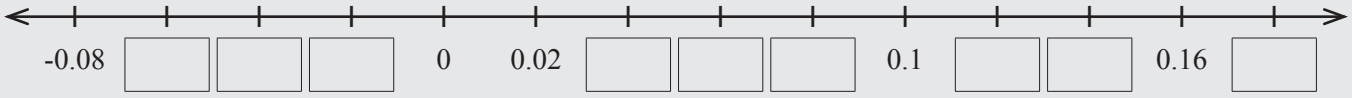
So $\sqrt{8}$ starts with 2 . ___ .

Complete these zoomed in number lines.

Ⓓ

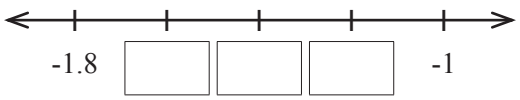


Ⓔ

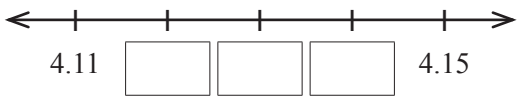


Where am I? Mark all possible solutions on each number line and write an algebraic expression to describe them.

Ⓕ Clue: I am greater than or equal to -1.2 _____



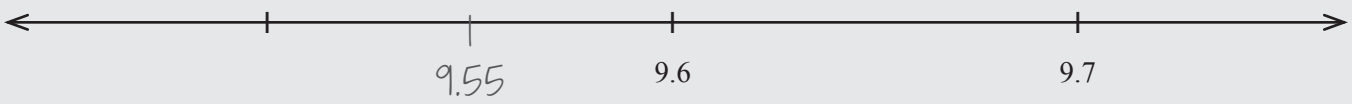
Ⓗ Clue: I am less than 4.13 _____



Ⓖ What number is *halfway* between -1.8 and -1? _____

Ⓘ What is the *distance* between 4.11 and 4.15? _____

Ⓙ Place these numbers on the number line below: 9.55 9.5 9.61 9.53 9.72 9.75 9.57 9.47



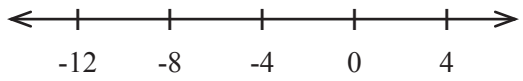
Ⓚ Which is greater, 9.6 or 9.52? _____

Ⓛ What number is exactly halfway between 9.47 and 9.57? _____

2-3 Where am I? Number Puzzles

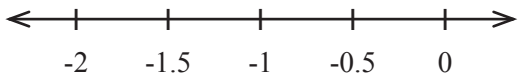
Where am I? Mark all possible solutions on each number line and write an algebraic expression to describe them.

① Clue: I am less than 4 _____



② What number is *halfway* between -8 and -4? _____

③ Clue: I am greater than or equal to -1 _____



④ What is the *distance* between -2 and -0.5? _____

⑤ Clue 1: I am less than 4
Clue 2: I am greater than or equal to _____

Ask yourself: Why isn't 4 a solution?

I am *x*. Where am I? $-1 \leq x < 4$. I could be 2 , ____ , ____ , ____ , ____ , or many other numbers.

Thinking out Loud

Michael , Lena, and Jay are looking at problem #5.

Michael: Wow, that's a complicated algebraic statement! Does that mean, "negative one is less than or equal to *x* and *x* is less than 4"?

Jay: It makes sense that you can just read it off the page like that, but I like to say it with the *x* first: "*x* is greater than or equal to -1."

Michael: What about the rest of it? What do you say about the 4?

Jay: Oh, well, for the whole thing, I say "*x* is greater than or equal to -1 and *x* is less than 4."

Lena: So, you can say it either way as long as it means the same thing. I like that!

Discussing Together

How could you read the algebraic statement $-6 < x \leq -2$?

Michael: So, what about writing - like this one: "*x* is less than 3 and *x* is greater than or equal to 1"?

Lena: Let's draw it on a number line! *(Lena draws a number line and marks the interval Michael described.)* Well, *x* is between 3 and 1... I would write the *x* first *(Lena writes* x *on the board)*, then the numbers around it, and the symbols last. 1 is less than 3 and to the left of 3 on the number line, so I write that on the left *(Lena writes 1 to the left and 3 to the right leaving space for symbols.)*

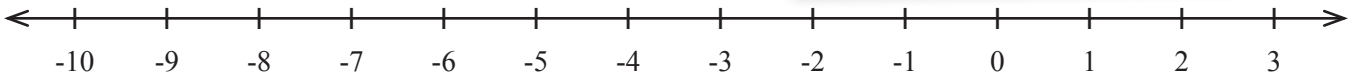
Michael: Oh! And then we know everything on the right is greater so we have to write the symbols this way! *(Michael completes the statement:* $1 \leq x < 3$)

Jay: We could write the 3 first also. The signs would just come out the other way! *(Jay writes* $3 > x \geq 1$*)*

Lena: Oh, Jay, you always have another way!

⑦ Clue 1: I am less than or equal to 0
Clue 2: I am greater than -3

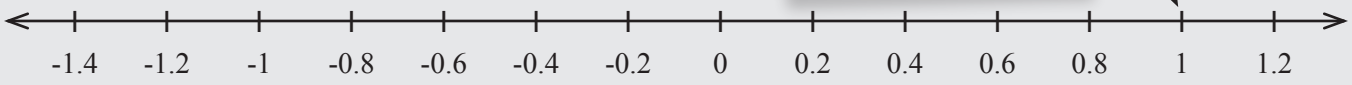
Mark the interval on the number line.



I am *x*. Where am I? _____ . I could be -1.2 , _____ , _____ , _____ , or many other numbers.
(write an algebraic statement)

⑧ Clue 1: I am less than or equal to 1
Clue 2: I am greater than -1

Is 1 a solution? Is -1?

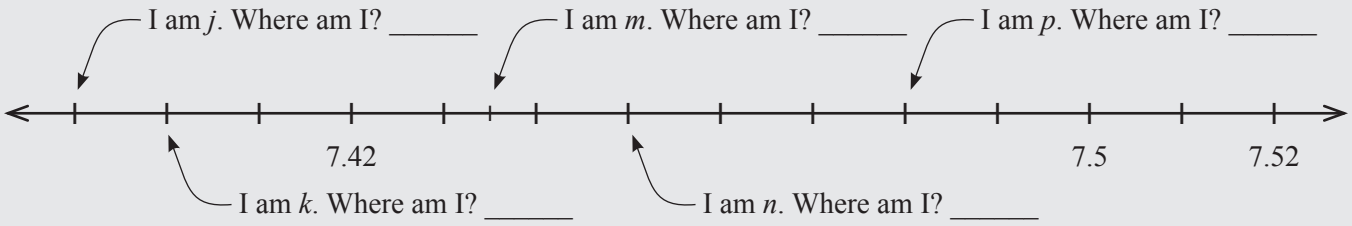


I am *x*. Where am I? _____ . I could be -0.6 , _____ , _____ , _____ , or many other numbers.
(write an algebraic statement)

⑨ What is the *distance* between 1 and -1? _____

⑩ What number is *halfway* between 0 and -1? _____

⑪



⑫ Complete the clues to match this interval and fill in the number line and all of the blanks.

Clue 1: I am less than _____

Clue 2: I am greater than _____

Is this number a solution? What does that tell you about the clue? And about the algebraic statement?

I am *x*. Where am I? _____ . I could be 0.3 , _____ , _____ , _____ , or many other numbers.
(write an algebraic statement)

⑬ What is the *distance* between 0.2 and 0.8? _____

⑭ What is the *distance* between -0.1 and 0.8? _____

⑮ Create your own puzzle! Write two clues with decimals, and swap puzzles with a partner to solve.

Clue 1:

Clue 2:

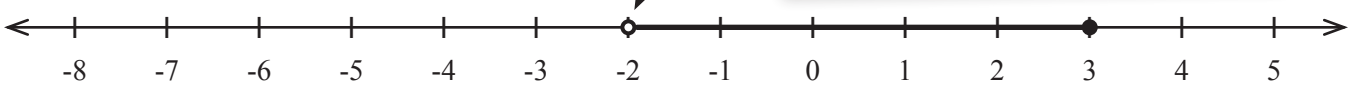
I am *x*. Where am I? _____ . I could be _____ , _____ , _____ , _____ , or many other numbers.

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

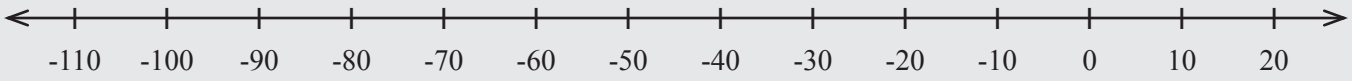
A Clue 1: I am greater than -2
Clue 2: I am less than or equal to _____

Ask yourself: Why isn't -2 a solution?



I am *x*. Where am I? $-2 < x \leq$ _____ (write an algebraic statement). I could be 0 , _____ , _____ , _____ , or many other numbers.

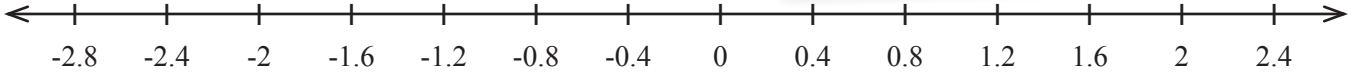
B Clue 1: I am greater than or equal to -110
Clue 2: I am less than 20



I am *x*. Where am I? _____ (write an algebraic statement). I could be _____ , _____ , _____ , _____ , or many other numbers.

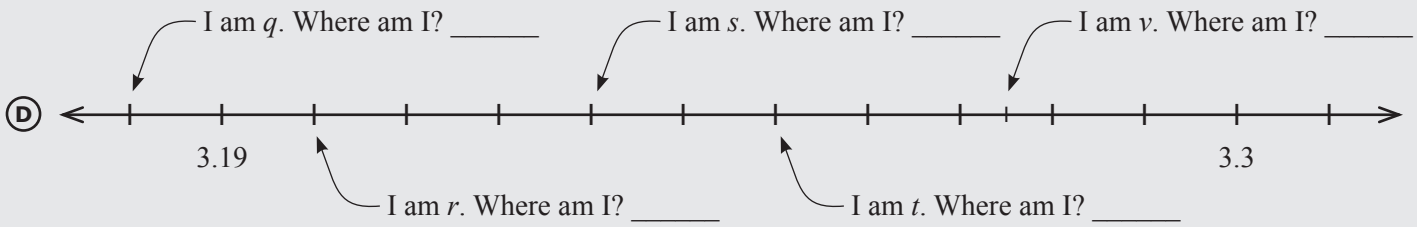
C Clue 1: I am greater than or equal to -2
Clue 2: I am less than 2

Is 2 a solution? Is -2?



I am *x*. Where am I? _____ (write an algebraic statement). I could be 1.2 , _____ , _____ , _____ , or many other numbers.

D



E Clue 1: I am less than _____

Clue 2: I am greater than _____

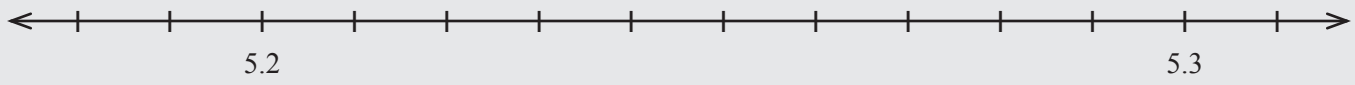
I am *x*. Where am I? _____ (write an algebraic statement). I could be 1.03 , _____ , _____ , _____ , or many other numbers.

F What is the *distance* between 1 and 1.09? _____

G What is the *distance* between 1.01 and 1.08? _____

2-4 Zooming in Differently

① Clue 1: I am less than 5.3
Clue 2: I am greater than 5.22



I am *x*. Where am I? _____ . I could be 5.24 , _____ , _____ , _____ , or many other numbers.
(write an algebraic statement)

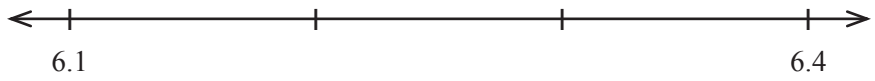
② What number is exactly halfway between 5.2 and 5.3? _____

③ Which is greater, 5.3 or 5.24? _____

④ What number is exactly halfway between 6.2 and 6.3? _____

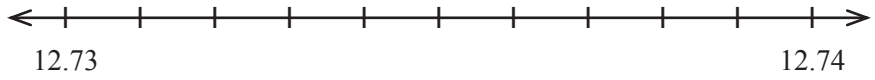
⑤ Which is greater, 7.18 or 7.3? _____

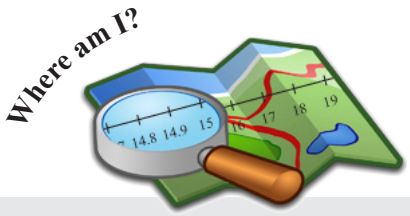
⑥ What number is exactly halfway between 6.1 and 6.4? _____



Don't be tricked by these two. Use the number lines to figure them out!

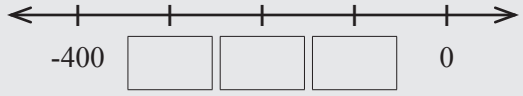
⑦ **Challenge!** What number is exactly halfway between 12.73 and 12.74? _____





⑧ Clue: I am greater than -300

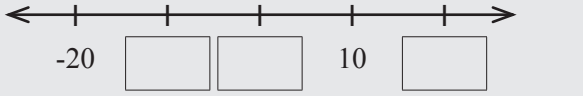
x ___ -300



⑨ What is *halfway* between -300 and -200? _____

⑩ Clue: I am less than or equal to 10

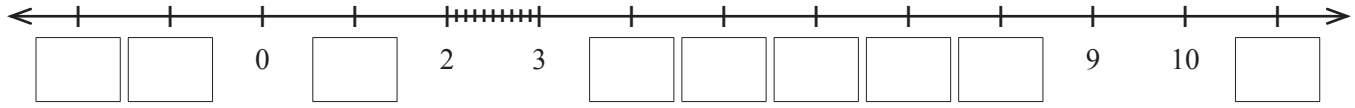
_____



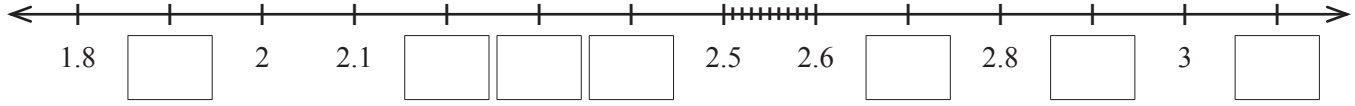
⑪ What is the *distance* between -10 and 20? _____

Complete these **Zooming In** number lines.

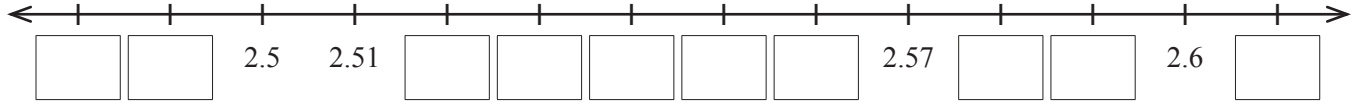
⑫



⑬



⑭



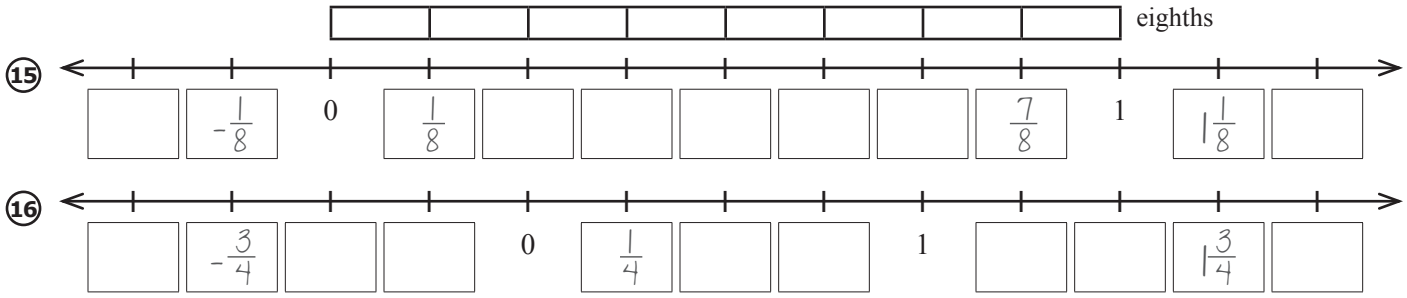
Algebraic Habits of Mind: Using Tools Strategically

The number line is a tool for reasoning about order and operations with real numbers.

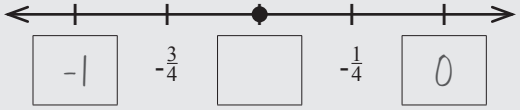
You can zoom in and zoom out on the number line to see multiples of 10 and 100 or tenths and hundredths.

Zooming in to see tenths and hundredths is like dividing the number line up by 10 again and again.

You can divide up the number line in other ways too...

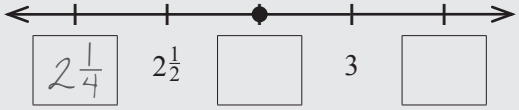


17 Clue: I am halfway between $-\frac{3}{4}$ and $-\frac{1}{4}$ X = ___



18 What is the *distance* between -1 and $-\frac{1}{4}$? _____

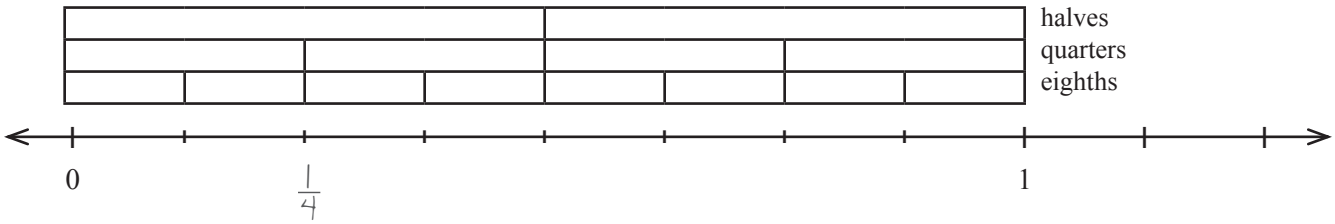
19 Clue: I am $\frac{1}{2}$ of the way from $2\frac{1}{2}$ to 3 X = ___



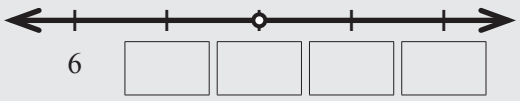
20 What number is *halfway* between $2\frac{1}{4}$ and $3\frac{1}{4}$? _____

21 Use the picture as a reference as you fill in this number line with the numbers below. Cross each one out as you go.

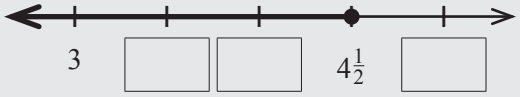
$\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{8}$ $\frac{3}{8}$ $1\frac{1}{8}$ $\frac{5}{8}$ $\frac{1}{2}$ $1\frac{1}{4}$ $\frac{7}{8}$



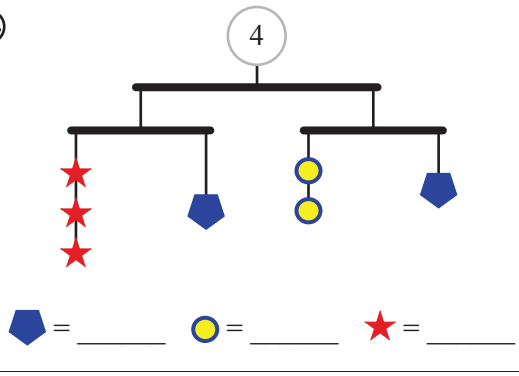
22 Clue: I am not 6.2 X ___ 6.2



23 Clue: I am less than or equal to $4\frac{1}{2}$ X ___ $4\frac{1}{2}$



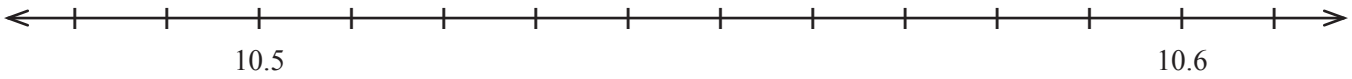
24



Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

A Clue 1: I am less than 10.5
Clue 2: I am greater than 10.6

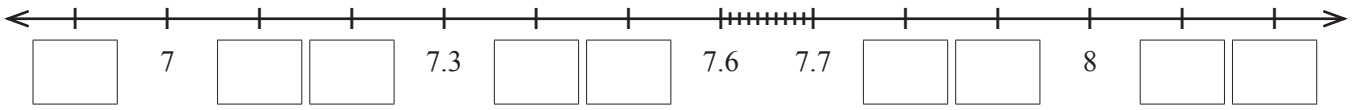


I am *x*. Where am I? _____ . I could be 10.56 , _____ , _____ , or many other numbers.
(write an algebraic statement)

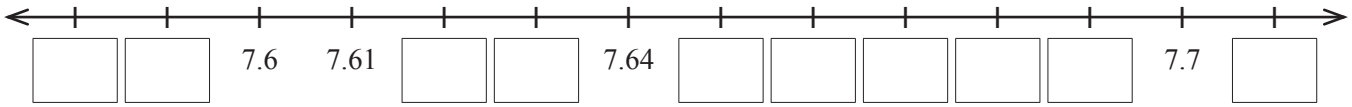
B What number is exactly halfway between 10.52 and 10.58? _____

C Which is greater, 10.6 or 10.48? _____

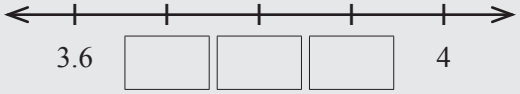
D



E

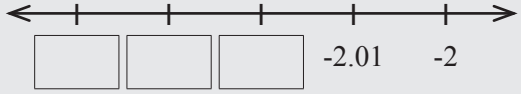


F Clue: I am less than or equal to 3.7 ✕ ___ 3.7



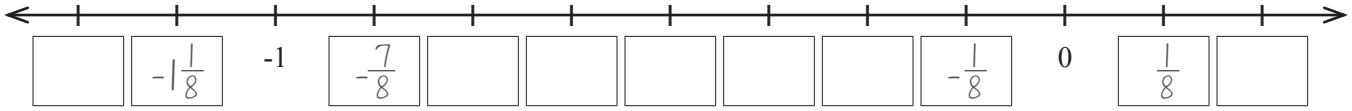
G What number is *halfway* between 3.6 and 4? _____

H Clue: I am greater than -2.03 _____

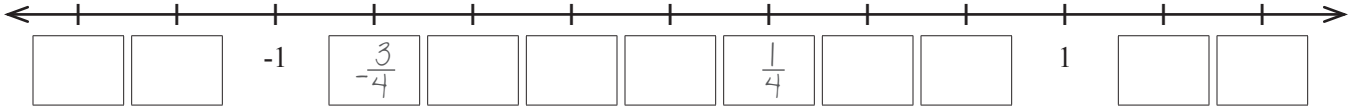


I What is the *distance* between -2 and -2.02? _____

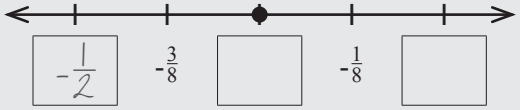
J



K

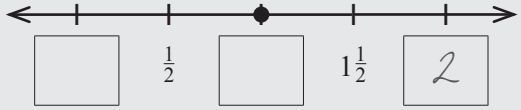


L Clue: I am halfway between $-\frac{3}{8}$ and $-\frac{1}{8}$ ✕ = ___



M What is the *distance* between 0 and $-\frac{3}{8}$? _____

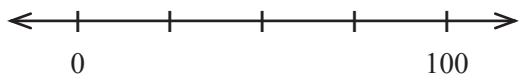
N Clue: I am $\frac{1}{2}$ of the way from $\frac{1}{2}$ to $1\frac{1}{2}$ ✕ = ___



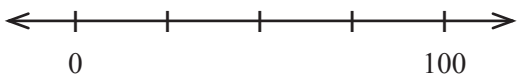
O What number is *halfway* between -1 and 0? _____

2-5 Subdividing the Number Line

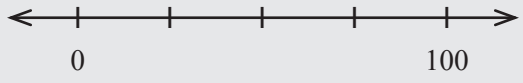
① Clue: I am $\frac{1}{4}$ of the way from 0 to 100



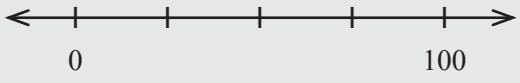
② Clue: I am $\frac{1}{4}$ of the way from 100 to 0



③ Clue: I am $\frac{3}{4}$ of the way from 0 to 100



④ Clue: I am $\frac{3}{4}$ of the way from 100 to 0



Discussing Together Some of the problems above had the same answer. Why?

For these problems, write your answers as **improper fractions**.

⑤ What is half of 7? $\frac{7}{2}$

⑥ What is half of 13? _____

⑦ What is half of ◆? _____

⑧ What is a quarter of 7? $\frac{7}{4}$

⑨ What is a third of 11? _____

⑩ What is a third of x? _____

⑪ What is a fifth of 7? _____

⑫ What is *two* fifths of 7? $\frac{2 \cdot 7}{5}$

⑬ What is *two* fifths of x? _____

⑭ What is 2★ + 4★? _____

⑮ What is *two* fifths plus four fifths? _____

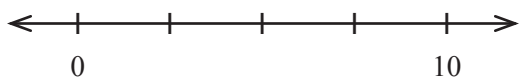
Use this number line with shapes to answer the questions below.

⑯ What is $\frac{2◆}{5} + \frac{4◆}{5}$? _____

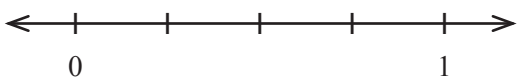
⑰ What is $\frac{◆}{5} + \frac{4◆}{5}$? _____

⑱ What is $\frac{3◆}{5} + \frac{7◆}{5}$? _____

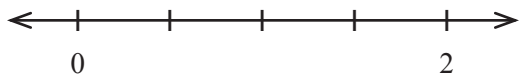
⑲ Clue: I am $\frac{1}{4}$ of the way from 0 to 10



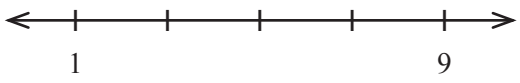
⑳ Clue: I am $\frac{1}{4}$ of the way from 0 to 1



㉑ Clue: I am $\frac{3}{4}$ of the way from 0 to 2

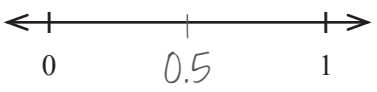


㉒ Clue: I am $\frac{3}{4}$ of the way from 1 to 9

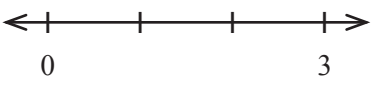


Write each fraction as a decimal and locate each number on the number line below it.

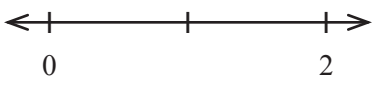
23 What is $1 \div 2$? $\frac{1}{2} =$ _____



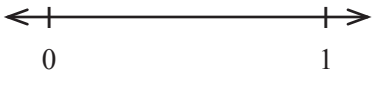
24 What is $3 \div 2$? $\frac{3}{2} =$ _____



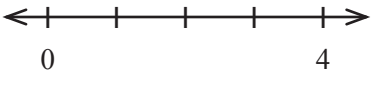
25 What is $2 \div 2$? $\frac{2}{2} =$ _____



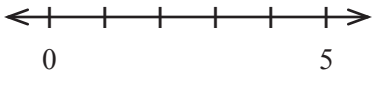
26 What is $1 \div 4$? $\frac{1}{4} =$ _____



27 What is $4 \div 2$? $\frac{4}{2} =$ _____

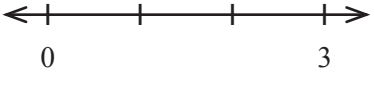


28 What is $5 \div 2$? $\frac{5}{2} =$ _____

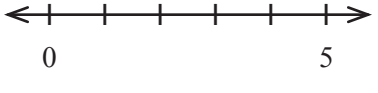


29 What is $1 \div 8$? $\frac{1}{8} =$ _____

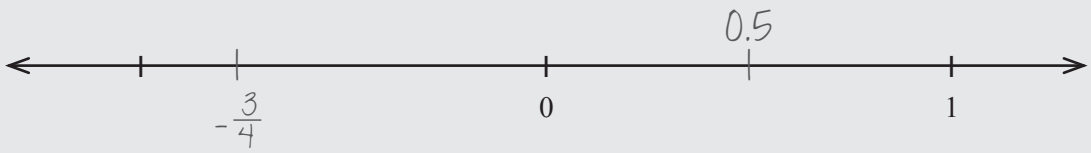
30 What is $3 \div 4$? $\frac{3}{4} =$ _____



31 What is $5 \div 4$? $\frac{5}{4} =$ _____



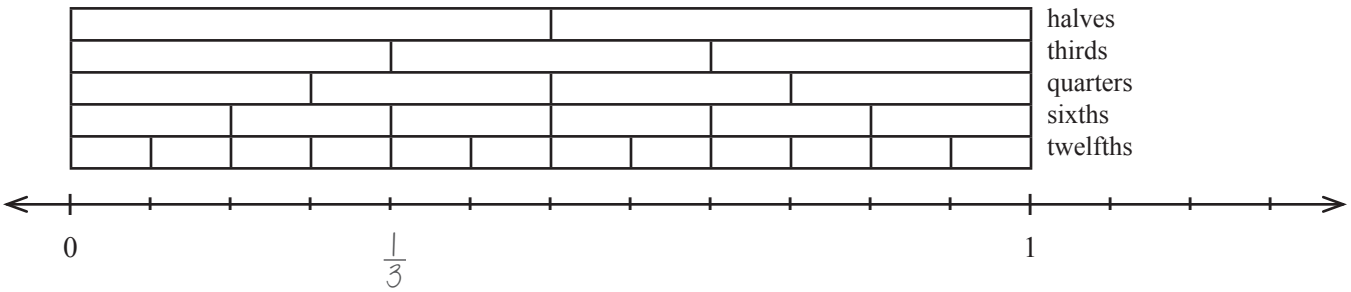
32 Place these numbers on the number line below: ~~0.5~~ $-\frac{3}{4}$ -1 $\frac{1}{2}$ 0.25 $\frac{3}{4}$ $-\frac{1}{2}$ -0.25 $\frac{1}{4}$ 0.75 -0.5



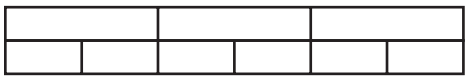
Discussing Together Are any of these numbers in the same locations? Which? Why?

33 Use the picture as a reference as you fill in this number line with the numbers below. Cross each one out as you go.

~~$\frac{1}{3}$~~ $\frac{3}{4}$ $\frac{1}{2}$ $\frac{1}{6}$ $\frac{7}{12}$ $\frac{5}{12}$ $\frac{1}{12}$ $1\frac{1}{12}$ $\frac{2}{3}$ $\frac{11}{12}$ $\frac{1}{4}$ $1\frac{1}{4}$ $\frac{5}{6}$ $1\frac{1}{6}$



34 Use the picture to explain why $\frac{1}{3} = \frac{2}{6}$.



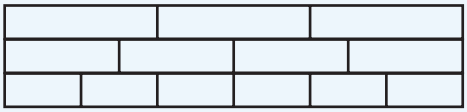
We can show one third by...

We can show two sixths by...

We know they are the same because...

35 Use the picture and whatever names you like to write a Thinking Out Loud dialogue with your group that explains why $\frac{2}{4} = \frac{3}{6}$. Practice it a few times together.

Thinking out Loud



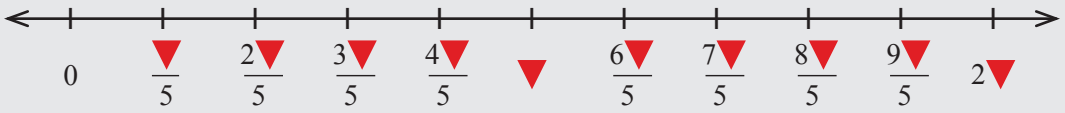
Algebraic Habits of Mind: Communicating Clearly

Reading the *Thinking out Loud* dialogues is a chance to practice speaking mathematically. This page is an opportunity to *create* that mathematical language yourself. Make sure your script communicates your ideas clearly before your present it.

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

Use this number line with shapes to answer the questions below.



(A) What is $\frac{4▼}{5} + \frac{3▼}{5}$? _____

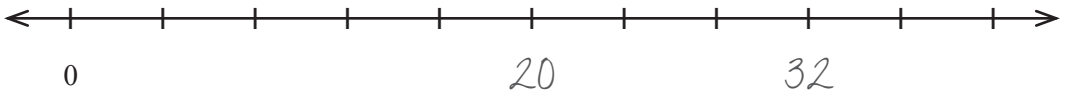
(B) What is $2 \cdot \frac{3▼}{5}$? _____

(C) What is $\frac{8▼}{5} + \frac{3▼}{5}$? _____

(D) What is $\frac{4▼}{5} - \frac{2▼}{5}$? _____

(E) What is $\frac{4▼}{5} - \frac{2▼}{5}$ when ▼ = 20? _____

(F) Rewrite the number line above when ▼ = 20.



Write each fraction as a decimal and locate each number on the number line below it.

(G) What is $1 \div 2$? $\frac{1}{2} =$ _____

(H) What is $1 \div 5$? $\frac{1}{5} =$ _____

(I) What is $2 \div 5$? $\frac{2}{5} =$ _____

(J) What is $3 \div 5$? $\frac{3}{5} =$ _____

(K) What is $2 \div 10$? $\frac{2}{10} =$ _____

(L) What is $4 \div 5$? $\frac{4}{5} =$ _____

(M) What is $6 \div 5$? $\frac{6}{5} =$ _____

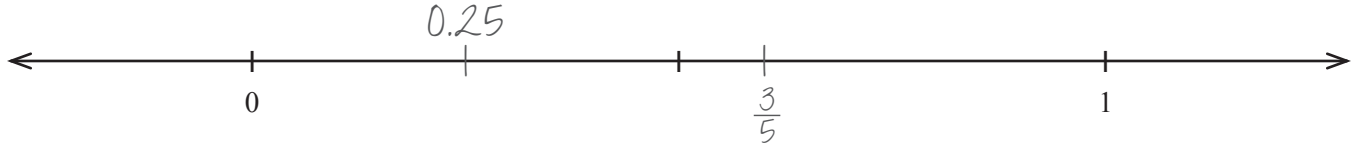
(N) What is $1 \div 10$? $\frac{1}{10} =$ _____

(O) What is $-1 \div 5$? $-\frac{1}{5} =$ _____

(P) Match them up! Match each fraction on the top with its equivalent decimal on the bottom.

$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{4}$	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{1}{4}$	$\frac{3}{5}$
0.2	0.5	0.8	0.25	0.4	0.75	0.6

(Q) Plot each fraction and decimal from the problem above on the number line below.



2-6 Ordering Logically

① Arrange $\frac{2}{3}$, $\frac{3}{7}$, and $\frac{2}{7}$ from least to greatest.

Thinking out Loud

Michael , Lena, and Jay are working on problem #1.

Lena: I know what to do! We have to get a common denominator. Let's see, 3 and 7 both go into 21...

Jay: Wait, maybe there is an easier way! Let's imagine where these fractions live on the number line.

Michael: Oh, can we compare them to $\frac{1}{2}$? *(He pauses.)* Yeah! $\frac{2}{7}$ and $\frac{3}{7}$ are less than $\frac{1}{2}$ because 2 and 3 are less than half of 7. And $\frac{2}{3}$ is more than $\frac{1}{2}$ because 2 is more than half of 3. So, $\frac{2}{3}$ is the biggest!

Pausing to Think How does Michael know that $\frac{2}{3}$ is the biggest?

Lena: Of course, and I can tell the other two. Since $\frac{3}{7}$ is more sevenths than $\frac{2}{7}$, $\frac{3}{7}$ has to be bigger!

② Arrange $\frac{4}{5}$, $\frac{1}{4}$, and $\frac{3}{5}$ from least to greatest.

③ Explain your reasoning to problem #2.

If I compare them all to $\frac{1}{2}$, I can tell that...

Then, when I look at the two that are left, I see that...

④ Arrange $\frac{5}{9}$, $\frac{2}{5}$, and $\frac{2}{9}$ from least to greatest.

Thinking out Loud

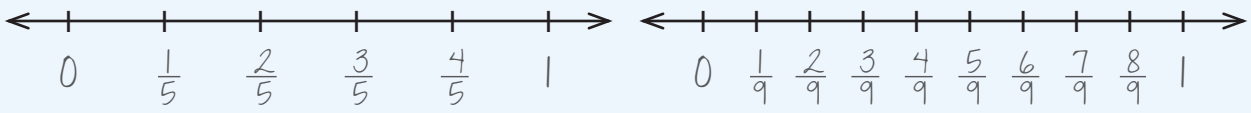
Now, Michael , Lena, and Jay are working on problem #4.

Michael: Wait! With this one, we can tell that $\frac{5}{9}$ is the biggest because it's the only one bigger than $\frac{1}{2}$, but what about the other two? $\frac{2}{5}$ and $\frac{2}{9}$ have different denominators so we cannot compare them!

Lena: But they have the same *numerator* on the top, so we can compare fifths and ninths. One fifth is 1 divided by 5, and one ninth is 1 divided by 9, so...

Michael: ...so ninths are smaller than fifths! That means $\frac{2}{9}$ is smaller than $\frac{2}{5}$.

Jay: I picture it in my head just like Lena described. *(Jay draws the interval 0 to on 1 on the board twice and divides them up with fifths and ninths.)* So I can see in my head why fifths are bigger!



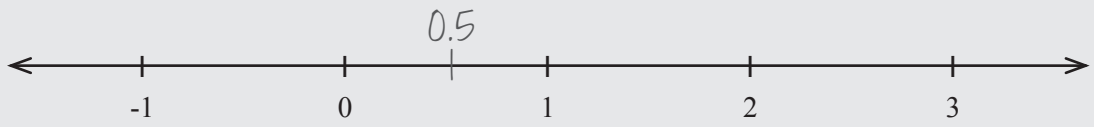
Circle the fraction with the greatest value.

⑤ $\frac{3}{9}$ or $\frac{3}{4}$ ⑥ $\frac{3}{9}$ or $\frac{4}{9}$ ⑦ $\frac{3}{4}$ or $\frac{1}{2}$ ⑧ $\frac{4}{9}$ or $\frac{1}{2}$ ⑨ $\frac{3}{4}$ or $\frac{4}{9}$

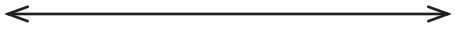
⑩ Arrange $\frac{3}{4}$, $\frac{3}{9}$, and $\frac{4}{9}$ from least to greatest.

⑪ Explain your reasoning to problem #10.

⑫ Place these numbers on the number line below: 0.5 - 1.8 $\frac{3}{4}$ 2.5 $\frac{1}{8}$ $-\frac{1}{4}$ -0.75 $1\frac{1}{4}$ $\sqrt{9}$

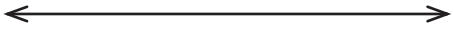


⑬ How far away from 1 is $\frac{7}{8}$?



Use a number lines to visualize these problems.

⑭ How far away from 1 is $\frac{8}{9}$?



Think about comparing these to 1.

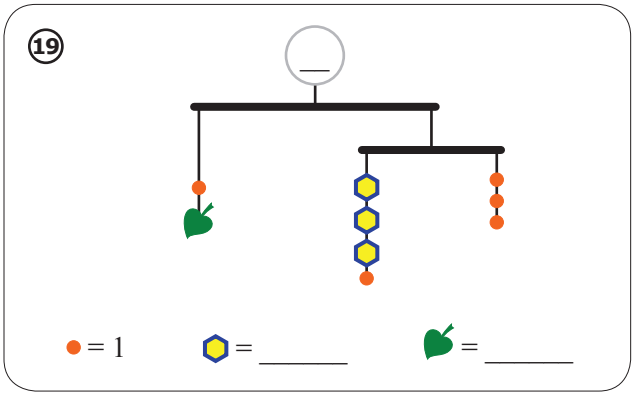
⑮ Which fraction has a greater value, $\frac{7}{8}$ or $\frac{8}{9}$? How do you know?

⑯ Which fraction has a greater value, $\frac{7}{8}$ or $\frac{5}{6}$? How do you know?

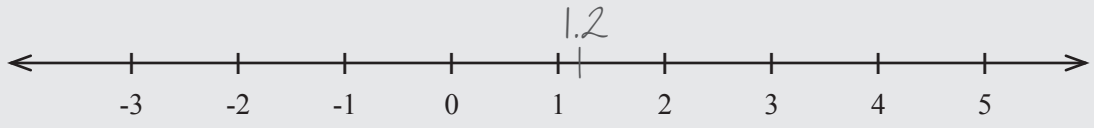
⑰ Arrange $\frac{5}{8}$, $\frac{3}{7}$, and $\frac{3}{8}$ from least to greatest.

⑱ Arrange $\frac{5}{8}$, $\frac{3}{7}$, and $\frac{3}{8}$ from least to greatest.

⑲



⑳ Place these numbers on the number line below: 1.2 $-2\frac{1}{2}$ $-\frac{4}{5}$ 2.6 $4\frac{1}{3}$ $-\frac{2}{3}$ -1.75 $1\frac{3}{4}$ $\sqrt{8}$



Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

A Which fraction has a greater value $\frac{5}{7}$ or $\frac{2}{7}$? _____
How do you know?

B Which fraction has a greater value $\frac{2}{5}$ or $\frac{2}{7}$? _____
How do you know?

Think about comparing to $\frac{1}{2}$.

C Which fraction has a greater value $\frac{2}{5}$ or $\frac{5}{7}$?
How do you know?

D Arrange $\frac{2}{5}$, $\frac{2}{7}$, and $\frac{5}{7}$ from least to greatest.

E Explain how you know if a positive fraction is greater than $\frac{1}{2}$ or less than $\frac{1}{2}$.

Think about $\frac{5}{7}$. Is that greater or less than $\frac{1}{2}$? How can you tell? Now describe how to tell for *any* fraction.

F How far away from 1 is $\frac{4}{5}$?

G How far away from 1 is $\frac{6}{7}$?

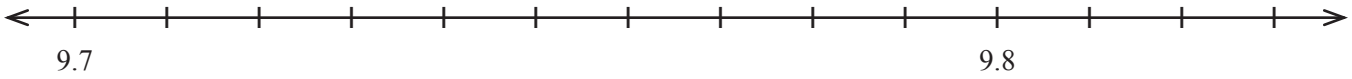
Think about comparing to 1.

H Which fraction has a greater value $\frac{4}{5}$ or $\frac{6}{7}$? How do you know?

I Arrange $\frac{4}{9}$, $\frac{7}{11}$, and $\frac{4}{11}$ from least to greatest. Explain your reasoning.

2-7 Revenge of the Where Am I? Number Puzzles

① Clue 1: I am less than or equal to 9.83
Clue 2: I am greater than 9.74



I am *x*. Where am I? _____ . I could be 9.76 , _____ , _____ , or many other numbers.
(write an algebraic statement)

② What number is exactly halfway between 9.7 and 9.8? _____

③ Which is greater, 9.8 or 9.77? _____

First, mark my interval on the number line. Then write two clues that would help find me.

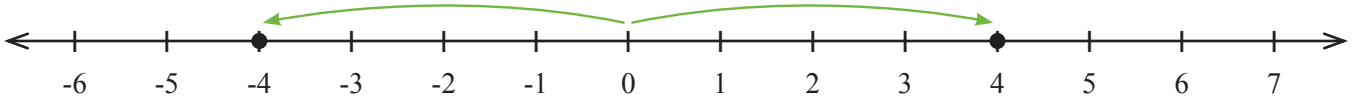
④ I am *x*. I am here: $-5 < x \leq 4$.

Clue 1:

Clue 2:

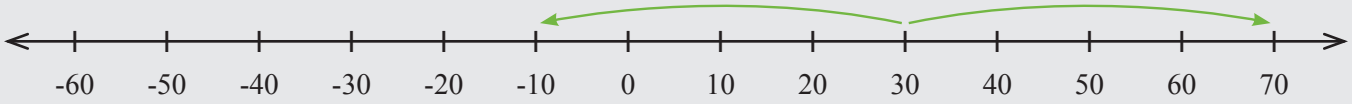
Where am I? Mark all possible solutions and fill in all of the blanks.

⑤ Clue: The distance between me and 0 is 4 $|x| = 4$



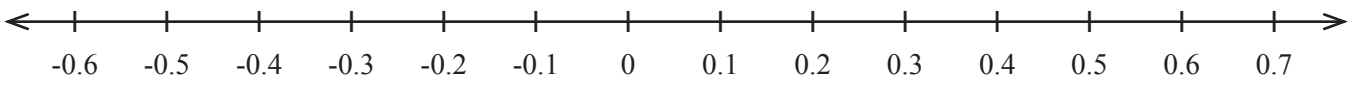
I am *x*. Where am I? I could be -4 or _____ .

⑥ Clue: The distance between me and 30 is 40 $|x - 30| =$ ____



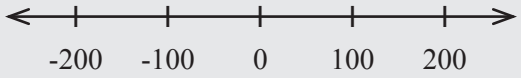
I am *x*. Where am I? I could be 70 or _____ .

⑦ Clue: The distance between me and 0.3 is 0.2 $|x -$ ____ $| = 0.2$

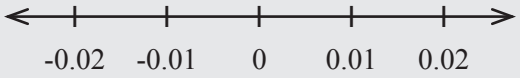


I am *x*. Where am ? I could be _____ or _____ .

⑧ Clue: I am negative _____



⑨ Clue: I am *not* negative _____

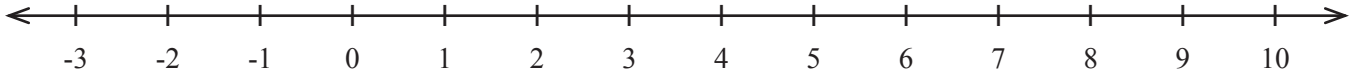


Where Am I? Mark all possibilities for each clue on the number lines, and write the algebraic expressions to match.

⑩ Clue 1: The distance between me and 4 is 1

$|x - 4| = 1$

Clue 2: I am closer to 1 than to 7



I am *x*. Where am I? I am 3 .

Discussing Together

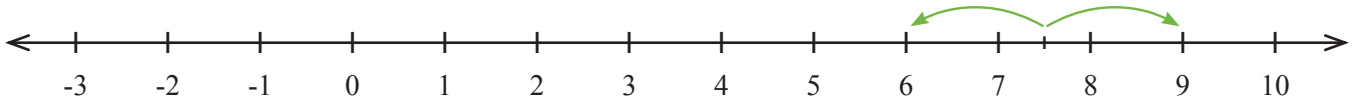
Why is there only one answer to problem #10?

How did you use the two clues to narrow down the possibilities?

⑪ Clue 1: The distance between me and $7\frac{1}{2}$ is $1\frac{1}{2}$

$|x - -7\frac{1}{2}| = 1\frac{1}{2}$

Clue 2: I am closer to 8 than to 5

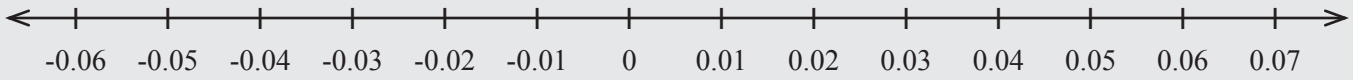


I am *x*. Where am I? I am _____ .

⑫ Clue 1: The distance between me and -0.02 is 0.04

$|x + 0.02| = 0.04$

Clue 2: I am closer to -1 than to 0



I am *x*. Where am I? I am _____ .

Where am I? Create your own puzzle! Write two clues with decimals or fractions, and give it to someone else to solve.

⑬ Clue 1:

Clue 2:

I am *x*. Where am I? _____ . I could be _____ , _____ , _____ , _____ , or many other numbers.

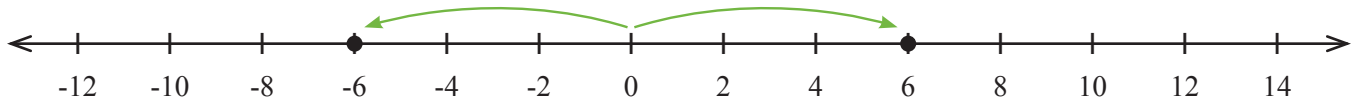
Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

Where am I? Mark all possible solutions and fill in all of the blanks.

A Clue: The distance between me and 0 is 6

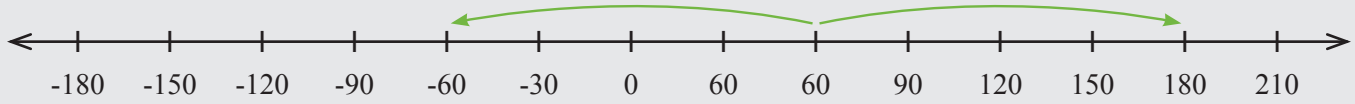
$|x| = 6$



I am *x*. Where am I? I could be -6 or _____ .

B Clue: The distance between me and 60 is 120

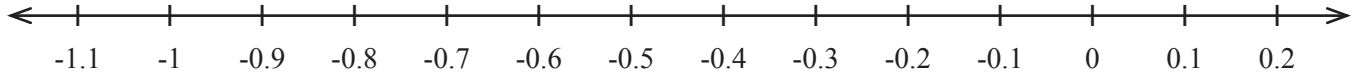
$|x - 60| =$ ____



I am *x*. Where am I? I could be _____ or _____ .

C Clue: The distance between me and -0.6 is 0.5

$|x -$ ____ $| = 0.5$



I am *x*. Where am I? I could be _____ or _____ .

D Clue 1: The distance between me and $3\frac{1}{2}$ is $2\frac{1}{2}$

$|x - -3\frac{1}{2}| = 2\frac{1}{2}$

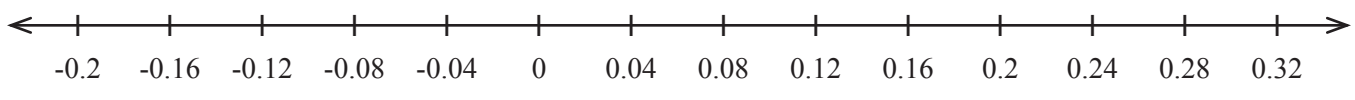
Clue 2: I am closer to 2 than to 5

I am *x*. Where am I? I am _____ .

E Clue 1: The distance between me and 0.04 is 0.12

$|x +$ ____ $| = 0.12$

Clue 2: I am closer to -0.12 than to 0

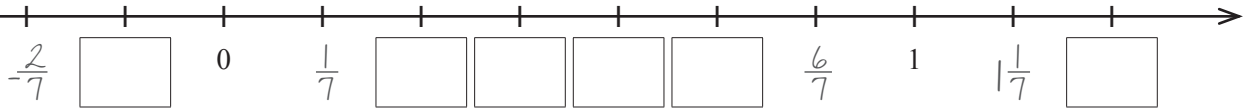


I am *x*. Where am I? I am _____ .

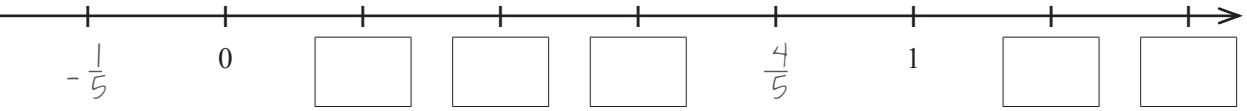
Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

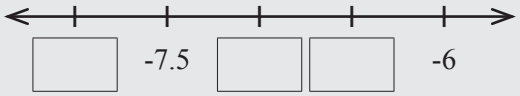
F



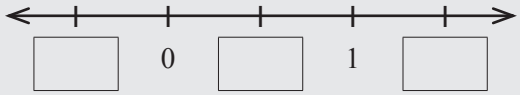
G



H Clue: I am not -6.5 _____



I Clue: I am greater than $\frac{1}{2}$ _____

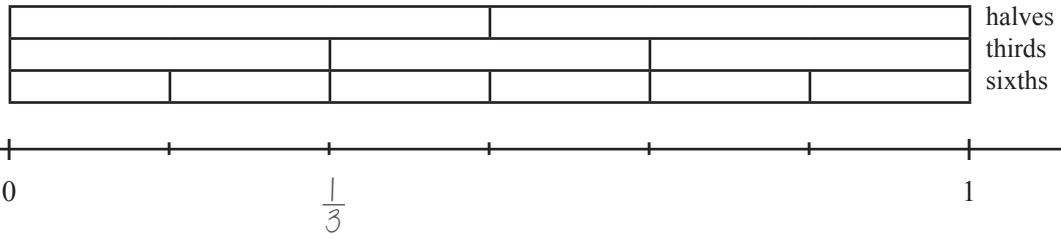


J What number is *halfway* between -7.5 and -6.5? _____

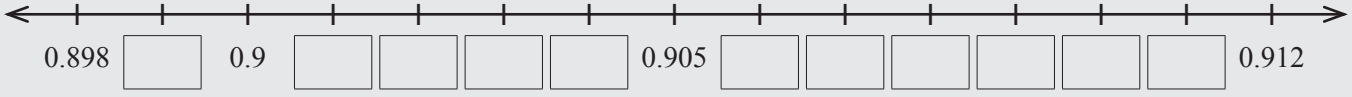
K What is the *distance* between $-\frac{1}{2}$ and $1\frac{1}{2}$? _____

L Use the picture as a reference as you fill in this number line with the numbers below. Cross each one out as you go.

$\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{6}$ $\frac{5}{6}$ $1\frac{1}{6}$

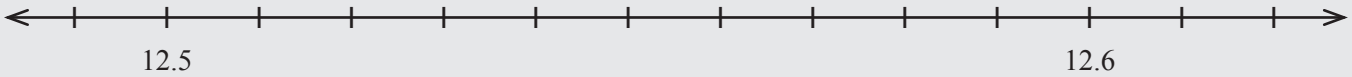


M



N Arrange $\frac{5}{6}$, $\frac{3}{4}$, and $\frac{5}{12}$ from least to greatest. Explain your reasoning.

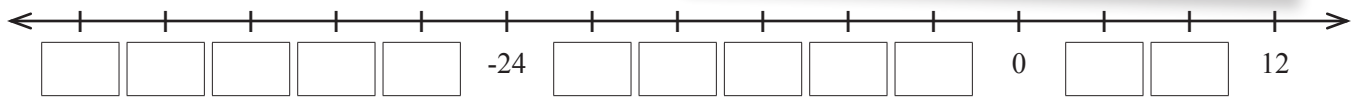
O Clue 1: I am less than 12.61
Clue 2: I am greater or equal to than 12.54

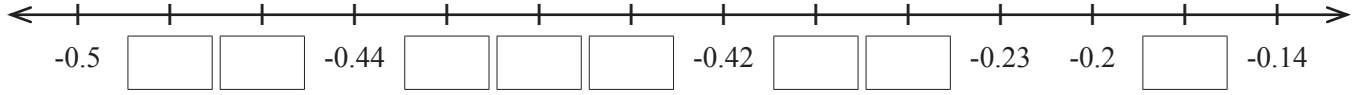


I am *x*. Where am I? _____ . I could be 12.57 , _____ , _____ , or many other numbers.
(write an algebraic statement)

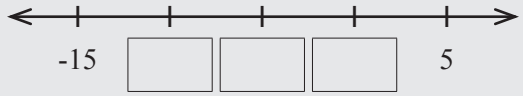
Unit Additional Practice Problems

Use this page to prepare for the unit exam.

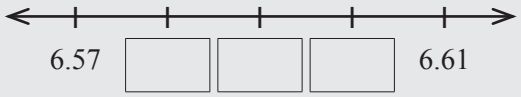
① 

② 

③ Clue: I am greater than or equal to -10 _____



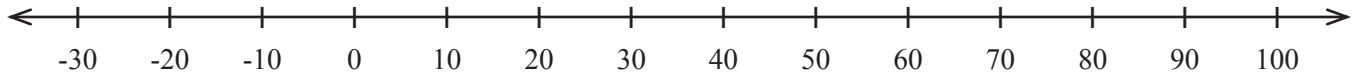
④ Clue: I am less than 6.59 _____



⑤ What is the *distance* between 5 and -15? _____

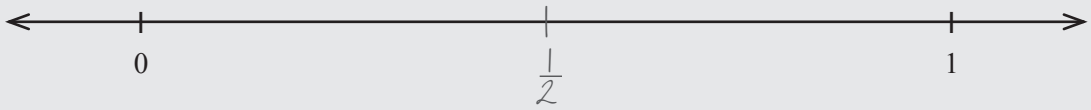
⑥ What number is *halfway* between 6.57 and 6.61? _____

⑦ Clue 1: I am greater than -20
Clue 2: I am less than or equal to 10



I am *x*. Where am I? _____ . I could be 5 , _____ , _____ , _____ , or many other numbers.
(write an algebraic statement)

⑧ Place these numbers on the number line below: ~~$\frac{1}{2}$~~ $\frac{3}{4}$ 0.5 $\frac{1}{8}$ 0.25 $\frac{3}{8}$ $\frac{1}{4}$ 0.75 $\frac{7}{8}$ 0.6 0.1



⑨ Arrange $\frac{3}{7}$, $\frac{2}{3}$, and $\frac{2}{7}$ from least to greatest. Explain your reasoning.

⑩ Clue 1: The distance between me and 5 is 4

$|x - 5| = 4$

Clue 2: I am closer to 2 than to 6

I am *x*. Where am I? I am _____ .

Unit 3: Distance and Sign

Who Am I?

So the last digit must be 0 or 5

- I am a multiple of 5.

h	t	u
		5

Aha! So u=5, and h+t = 10. So...

- $h + t = 2u$

u can't be 0, or h+t would also have to be 0...

- $h + u < t$

The only possibilities are: 195 ✓ 375 ✓ 555 735 915

- All of my digits are odd.
- My hundreds digit is less than my tens digit.

Transition to Algebra

Unit 3: Distance and Sign

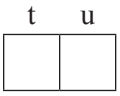
Walk twenty paces northeast from the old oak tree and dig three armlengths down.
There, the treasure will be...

Distance and direction are basic ideas in measurement, algebra, geometry, and, in fact, all of mathematics. The numbers 7 and -7 are very different numbers, but they are related because they are the same distance from 0. The sign tells you which direction from zero they are. You will use distances between numbers, like the distance between 4.9 and 5, to make sense of operations with negatives, decimals, fractions, and even algebraic expressions. In this unit, you will also meet "Who Am I?" puzzles, where you have to use clues to figure out the identity of a mystery number.

Lessons in this Unit:

Who Am I?

- I am odd.
- $u > t$
- The sum of my digits is 10.
- I am less than 30.

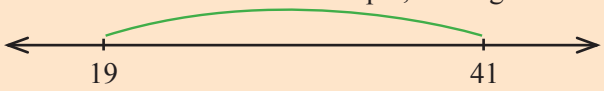


Can you find the mystery number?

Algebraic Habits of Mind: Using Tools

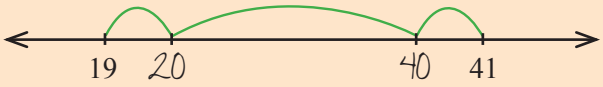
In Unit 2, you saw the structure of the number line. You saw where negative numbers live, and saw some numbers between numbers written as fractions and decimals. Now that you've seen the structure, Unit 3 will focus on using the number line as a tool.

There are many relationships we can examine with a number line. For example, we might be asked to find the distance between 19 and 41.



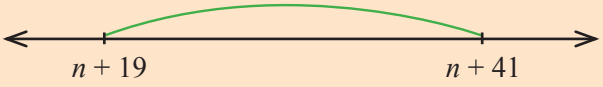
We can do this many different ways.

We could break the distance up into smaller, easier-to-find distances, and add them all up:



Or we could slide the numbers over to line up with some easier numbers:

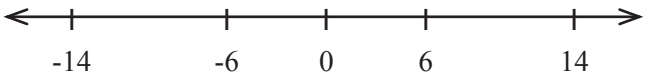
And did you know that the distance between 19 and 41, is the same distance between $n + 19$ and $n + 41$?



This works for *any* number you want to plug in for n. Try it!

 DRAFT

3-1 Making a Useful Number Line



① What is the distance between 6 and 14? _____

② What is the distance between -6 and -14? _____

③ How far apart are 6 and -14? _____

④ What about -6 and 14? _____

Algebraic Habits of Mind: Using Tools Strategically

When you sketch your own number line, you have a lot of choices to make.

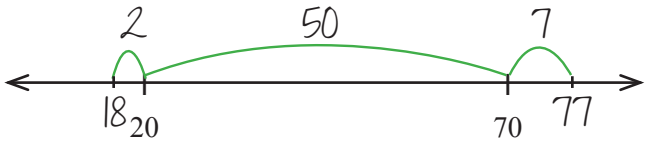
- What part of the number line will you draw? From -4.2 to -4.1? From 300 to 1000?
- What scale will you use? Mark every 1 unit? Every 0.01 units? Every 350 units?
- Often you don't need a scale at all, and you just need to keep track of a few important points.

Your choices depend on what you need your number line for.

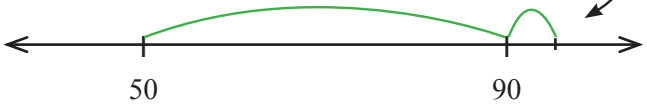
The scale you draw doesn't matter as much as having the correct idea in your head.

For each question, sketch a useful number line, and answer the question.

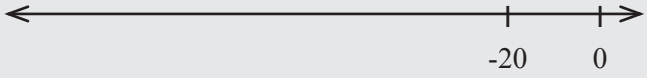
ex What is the distance between 18 and 77? 59



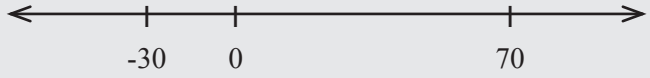
⑤ What's the distance between 91 and 50? _____



⑥ What's the distance between -20 and -79? _____



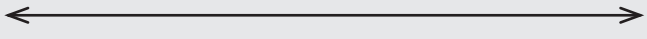
⑦ What about between -38 and 70? _____



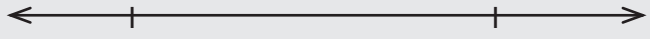
⑧ How far apart are -42 and -15? _____

⑨ How far apart are 96 and 402? _____

⑩ What about -8 and 501? _____



⑪ What about 2 and -5.1? _____



⑫ What about -8 and -1.9? _____

⑬ What about -3.2 and 7.6? _____

Algebraic Habits of Mind: Using Tools Strategically

You can draw a number line any time you want to, but sometimes it's enough just to imagine it. When you are familiar with what the numbers in a problem look like, it becomes easier to know what that distance should look like, too.

Now, draw a number line in your head (or on the page) to answer the questions below.

⑭ What is the distance between 60 and 140? _____

⑮ What is the distance between -10 and -70? _____

⑯ What's the distance between 50 and -300? _____

⑰ What about between -5 and 950? _____

⑱ What's the distance between 2.5 and -7.5? _____

⑲ How far apart are -20 and -45? _____

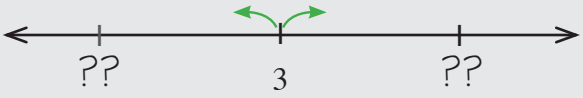
⑳ What about 2 and -5.3? _____

㉑ What about -4 and 3.54? _____

㉒ Visualize these two distances: from -10 to -30 and from -10 to 30.
How would you describe to a friend how the distances are different?

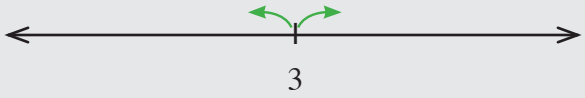
Share your answer with a friend.

㉓ Clue: The distance between me and 3 is 10



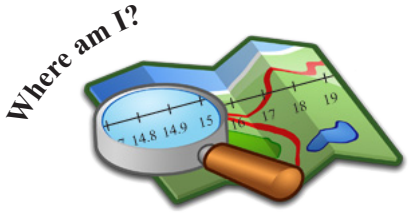
I am *x*. Where am I? I could be _____ or _____ .

㉔ Clue: I am 40 units away from 3



I am *x*. Where am I? I could be _____ or _____ .

㉕ I am *x*. I am here: $-5.3 < x \leq 4.5$. Show where I can be on the number line and write two clues that would help find me.



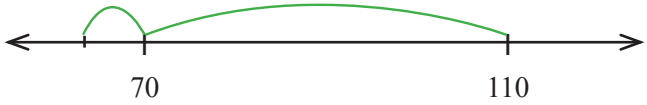
Clue 1:

Clue 2:

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.
For each question, sketch a useful number line and answer the question.

Ⓐ What is the distance between 63 and 110? _____



Ⓑ Find the distance between -30 and -82. _____

Ⓒ What's the distance between 42 and -17? _____

Ⓓ What about between -109 and -77? _____

Ⓔ How far apart are 1.4 and 9? _____

Ⓕ What about -3.6 and 5.8? _____

Sketch a number line in your head (or on paper) to answer the following.

Ⓖ What's the distance between 30 and 80? _____

Ⓗ What's the distance between -80 and 30? _____

Ⓘ What's the distance between -6 and 90? _____

Ⓙ What about between -74 and 10? _____

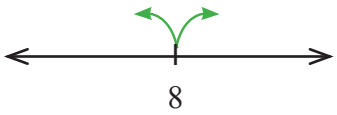
Ⓚ What about 402 and 100? _____

Ⓛ How far apart are -150 and -189? _____

Ⓜ How far apart are -201 and 50? _____

Ⓝ What about -505 and 505? _____

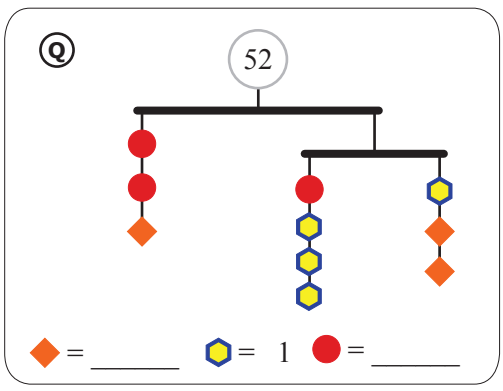
Ⓞ Clue: I am 20 units away from 8.



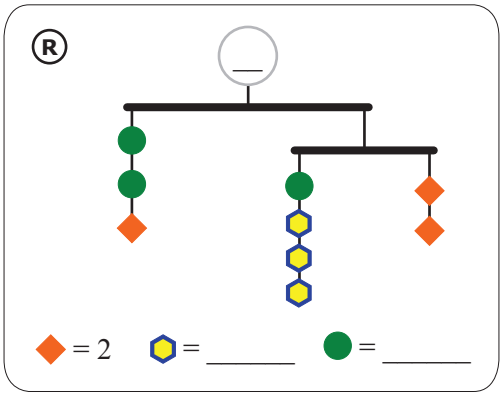
I am x. Where am I?

I could be _____ or _____ .

Ⓠ

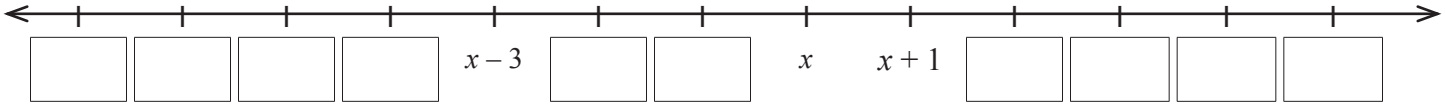


Ⓡ



3-2 Algebra on the Number Line

Fill in the number line and use it to answer the questions below.



① What is the distance between x and $x - 3$? 3

② What's the distance between $x + 2$ and $x - 4$? _____

③ What's the distance between $x - 1$ and $x + 1$? _____

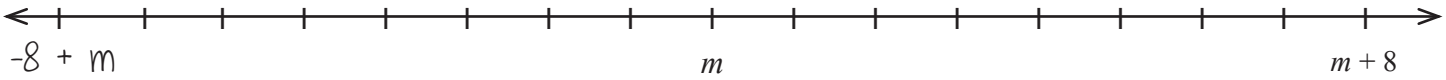
④ What about between $x - 6$ and $x - 1$? _____

⑤ Do problems #1-4 again using $x = 13$.
What if $x = -100$? Do all four answers stay the same?
Can you find a number for x that would change any of your answers?

Discuss your work with a friend. Test different numbers and see what happens.

⑥ Plot these values and label them on the number line below.

~~-8 + m~~ $m - 3$ $4 + m$ $2.5 + m$ $-1 + m$ $m + -5$ $m - -5$

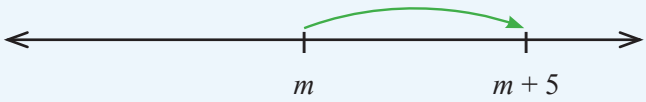


Thinking out Loud

Michael: $m - -5$ has just *got* to be the same as $m + 5$. I know that subtracting a negative is supposed to be like adding, and I can do it, but I really want it to make sense. Can we figure it out?

Lena: Um... Let's start with what we know. We know that $m + 5$ is larger than m.

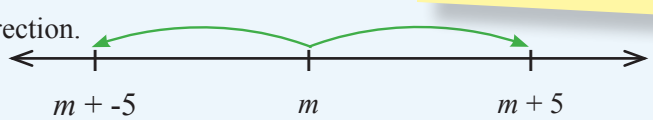
(Lena draws:)



There are no other marks on the number line, so $m + 5$ can be *anywhere* to the right of m.

Jay: And $m + -5$ is smaller than m because instead of adding 5, we're adding the opposite of 5, so we move in the other direction.

(Jay adds to Lena's picture:)

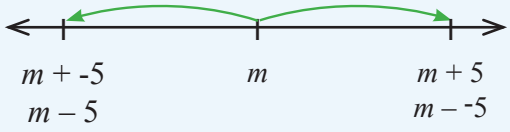


Michael: We also know that $m - 5$ is smaller than m, since we're subtracting 5 from whatever m is. It looks like $m - 5$ ends up in the same place as $m + -5$. They must be the same!

(Michael writes "m – 5" below Jay's "m + -5")

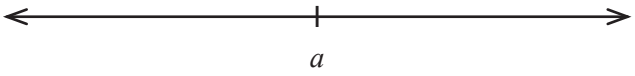
Lena: And so $m - -5$ has to be larger than m. We're subtracting the opposite of 5, so we move in the opposite direction from subtracting 5. We add 5!

(Lena writes "m – -5" below her original "m + 5")



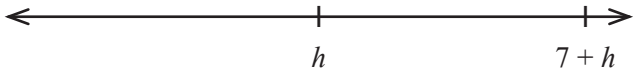
⑦ On this number line, a is labeled. Plot:

$a + 6$ $a + -6$ $a - 6$ $a - -6$



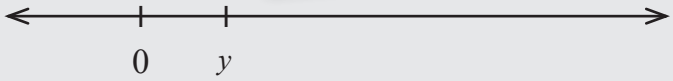
⑧ On this number line, h and $7 + h$ are labeled. Plot:

$2 + h$ $-2 + h$ $h + -7$ $h - -7$



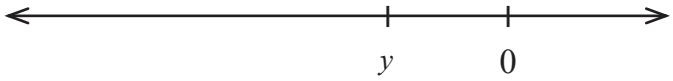
⑨ On this number line, y is labeled. Mark $2y$ and $5y$.

Mark off distances on the edge of a piece of paper to make your own ruler.



⑩ How did you decide where $5y$ goes in problem 9?

⑪ On this number line, y is labeled. Mark $3y$ and $-y$.

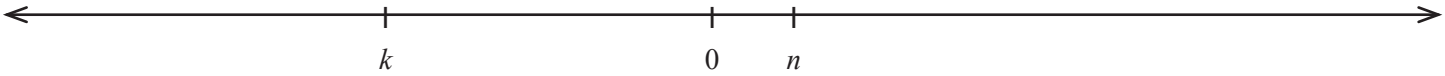


⑫ How did you decide where $-y$ goes in problem 11?

Share your answers to #10 and #12 with a friend.

⑬ Show where each of these belongs on the number line.

$5n$ $2k$ $-k$ $-2n$ $-2k$



Sketch a number line (here or in your head) to answer the questions below.

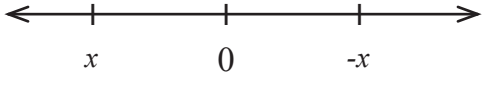
⑭ What is the distance between a and $a + 3$? _____

⑮ What's the distance between $x - 5$ and x? _____

⑯ What about between $x + 4$ and $x + 20$? _____

⑰ What about between $n - 2$ and $n + 30$? _____

⑱ At first, this number line looks wrong, because we know that negative *numbers* live to the left of 0 on the number line. But because x is a *variable*, it is possible for $-x$ to be a positive number. Use examples to explain this number line to a person seeing it for the first time.

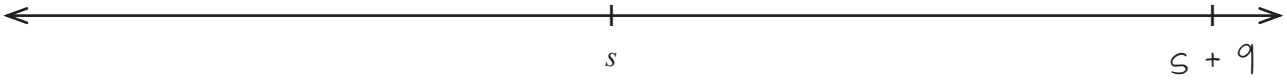


Additional Practice Problems

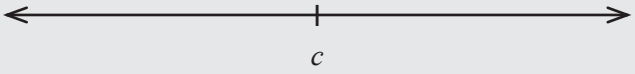
Select problems that will help you learn. Do some problems now. Do some later.

A Show where each of these belongs on the number line.

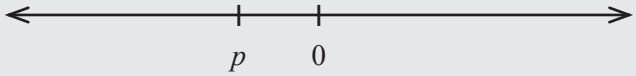
$s + 9$ $s - 4$ $s + -1$ $6.5 + s$ $-7 + s$ $s - -2$



B On this number line, c is labeled. Mark $c + 3$ and $c - -8$ and $-4 + c$

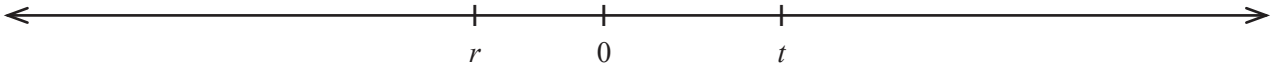


C On this number line, p is labeled. Mark $3p$ and $-2p$

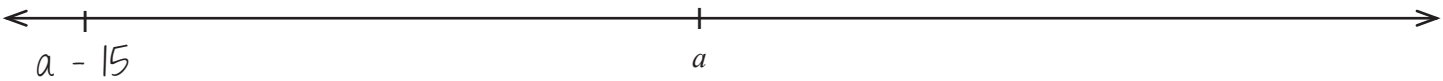


D Plot and label these values on the number line.

$2t$ $4r$ $3t$ $2r$ $-r$



E For each pair, figure out how far apart the two numbers are.



What is the distance between....

a and $a + 6$? _____

$a + 8$ and a? _____

a and $a - 10$? _____

$a - 4$ and a? _____

$a + 3$ and $a + 12$? _____

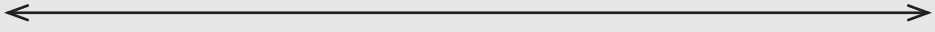
$a - 2$ and $a - 15$? _____

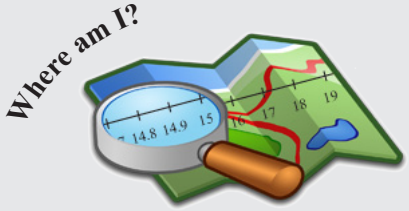
$a - 2$ and $a + 7$? _____

$a - 5$ and $a + 1$? _____

First, mark my interval on the number line. Then write two clues that would help find me.

F I am x. I am here: $-1.6 < x \leq -1.12$. I could be _____, _____, _____, or many other numbers.





Clue 1:

Clue 2:

3-3 Who Am I? Puzzles

① Who Am I?
- I am even.
- The sum of my digits is 16.

t u

0 2 4 6 8

② Who Am I?
- I am even.
- I am a perfect square.
- My units digit is twice my tens digit.

t u

0 + 4 9 = 16

③ Who Am I?
- I am odd.
- The sum of my digits is 2.

t u

④ Who Am I?
- I am a multiple of 10.
- I am between 4^2 and 5^2.

t u

⑤ Who Am I?
- I am even.
- The sum of my digits is 4.
- My units digits is not 2.

t u

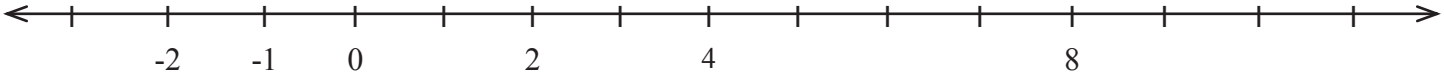
⑥ Who Am I?
- I am odd.
- $u = t$
- The sum of my digits is 6.

t u

Algebraic Habits of Mind: Puzzling & Persevering

To solve Who Am I? puzzles you have to play with the digits both separately *and* together to find the mystery number. When solving puzzles, always start with what you know. These clues will eventually help you narrow the answer, but there can be many ways to get there. Try anything that looks hopeful.

⑦ For each pair of numbers, figure out how far apart the two numbers are. Use the number line to help you.



What is the distance between....

3 and 7? _____ 8.5 and 10? _____ -3 and -1? _____

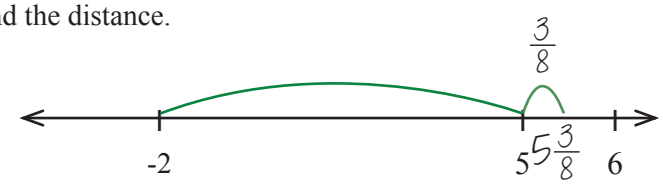
$-2\frac{1}{2}$ and -3? _____ $8\frac{1}{4}$ and 9? _____ $4\frac{1}{2}$ and 10? _____

$\frac{1}{2}$ and 1? _____ 8 and $9\frac{1}{4}$? _____ -3 and 12? _____

-3 and 7? _____ $8\frac{3}{4}$ and 9? _____ -3 and -3? _____

Label the numbers on the number line and use it to find the distance.

⑧ What is the distance between -2 and $5\frac{3}{8}$?



⑨ What's the distance between $7\frac{1}{3}$ and 12?

10 Answer **True** or **False** for each statement below. If the statement is false, give a specific example that that proves the statement is false.

$5n$ is always positive. False $-k$ is always negative. _____

Ex: If n is -3, 5n is _____

If a is a positive number, then $a + 10$ is always positive. _____

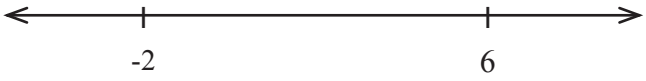
If m is a positive number, then $m - 8$ is always negative. False

Your specific example can be a number or a picture.

Since the variable can represent *any* number, all you need is *one* example to show that a statement is false.

11 What's the distance between $10\frac{4}{5}$ and 18?

12 Find the distance between $6\frac{1}{5}$ and $-2\frac{1}{5}$.



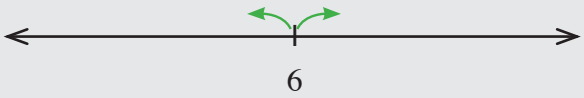
13 What's the distance between -13 and $-\frac{1}{4}$?

14 What about between $9\frac{3}{8}$ and 17?

15 Clue: The distance between me and 4 is $20\frac{1}{2}$

I am x. Where am I? I could be _____ or _____ .

16 Clue: I am $100\frac{1}{4}$ units away from 6



I am x. Where am I? I could be _____ or _____ .

17 Who Am I?

- Both of my digits are odd.
- My tens digit is 6 less than my units digit.
- I am less than 20.

t u

18 Who Am I?

- I am a perfect square.
- The sum of my digits is 9.
- My units digit is less than my tens digit.

t u

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

A Who Am I?

- Both of my digits are odd.
- Neither of my digits are perfect squares.
- I am one less than a perfect square.

t u

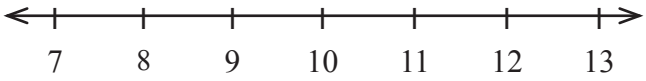
B Who Am I?

- The sum of my digits is 14.
- My digits are not the same.
- Both my digits are odd.
- I am less than 90.

t u

Label the two numbers on the number line and use it to find the distance.

C What is the distance between $8\frac{2}{5}$ and 13?



D What is the distance between $-5\frac{3}{4}$ and 1?

E What's the distance between $-9\frac{3}{5}$ and $-10\frac{1}{5}$?

F What's the distance between $10\frac{1}{6}$ and $1\frac{5}{6}$?

G What's the distance between $15\frac{5}{7}$ and $26\frac{1}{7}$?

H Who Am I?

- The sum of my digits is 9.
- My tens digit is even.
- My tens digit and units digit differ by 1.

t u

I Who Am I?

- The sum of my digits is 18.

t u

J Who Am I?

- Both of my digits are even.
- My tens digit is 6 more than my units digit.
- My digits add to 10.

t u

K Who Am I?

- My digits are the same.
- The sum of my digits is greater than 12 and less than 16.

t u

3-4 Making Change

Carla has exactly 22 cents in her pocket.

1. Malika, Jing, and Eva are writing down everything they can say for sure about the coins in Carla's pocket. Most of what they said is correct, but 4 of these statements are wrong. Cross out the incorrect statements.

She has no quarters.
She has to have exactly two dimes.
She could have two dimes.
She could have up to four nickels.
She could have three nickels.
She has to have exactly two pennies.
She has to have at least two pennies.
She has to have more than three coins.
The biggest number of coins she could have is 22.
She could have four coins.
She could have five coins.
She could have six coins.
She could have seven coins.
She could have 21 coins.

Share your answers with a friend.

2. How did you know that those four statements were wrong?

3. Ben has exactly 18 cents in his pocket.

   Write down at least ten different statements you could say for sure about the coins in Ben's pocket.

Compare your answers with someone else and check their answers.

4. Who Am I?
   - I am even.
   - The sum of my digits is 3.
   - The product of my digits is 2.

t	u

5. Who Am I?
   - Both of my digits are odd.
   - My digits add up to 12.
   - My tens digit is two more than my units digit.

t	u

6. Circle the fractions that are greater than $\frac{1}{2}$:

$\frac{1}{3}$ $\frac{4}{7}$ $\frac{2}{3}$ $\frac{1}{10}$ $\frac{3}{8}$ $\frac{5}{6}$

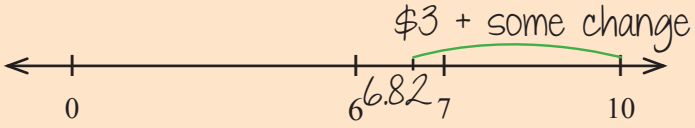
$\frac{4}{9}$ $\frac{3}{6}$ $\frac{2}{5}$ $\frac{1}{8}$ $\frac{5}{8}$ $\frac{3}{7}$

7. How do you know if a fraction is greater than $\frac{1}{2}$?

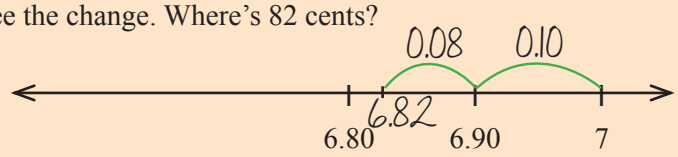
Algebraic Habits of Mind: Using Tools Strategically

The number line can even help you make change. Imagine you're a cashier at the corner store, but the cash register is broken. A customer spends $6.82 and pays with a $10 bill.

Picture your number line. Where is $6.82 on a $10 number line?



Then zoom in to see the change. Where's 82 cents?



You owe the customer $3.18 in change.

We write this process using subtraction. After you take away $6.82 from $10, what's left?

10 - 6.82 = 3.18

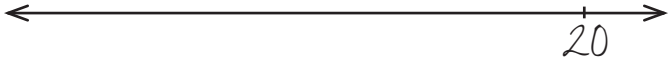
8 Asher's total purchase is $4.87, and he pays with a $10 bill. How much change should you give?

10 - 4.87 = _____

9 Kayla's purchases total $7.56, and she pays with a $10 bill. How much change should you give?

10 - _____ = _____

10 Paulo spent $11.14, and he pays with a $20 bill. How much change should you give?

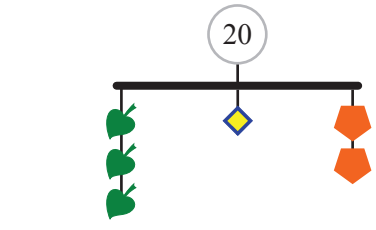


_____ - _____ = _____

11 Sang's total purchase is $2.73, and she pays with a $5 bill. How much change should you give?

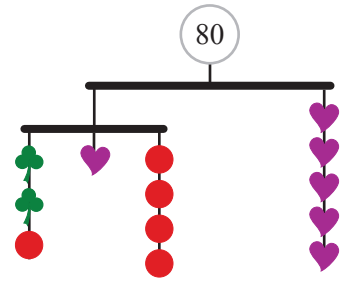
_____ - _____ = _____

12



◇ = _____ ⬟ = 3 🍃 = _____

13



♥ = _____ ● = 4 ♣ = _____

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

A A book costs $14.98. The customer hands the clerk a $20 bill. How much change should the customer get?

_____ - _____ = _____

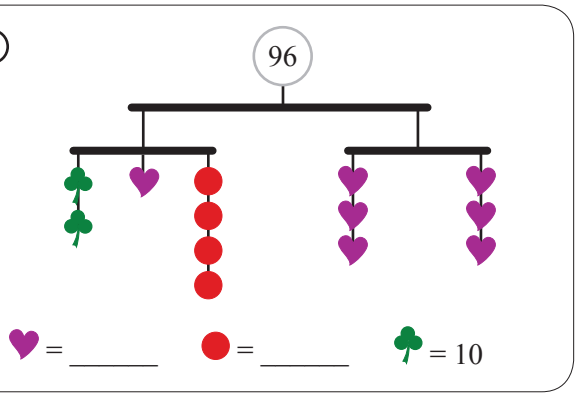
B Lunch and tip cost $6.57, but I only had a $20 bill. How much change should the waiter return?

_____ - _____ = _____

C Your purchases cost $13.24, and you paid with a $50 bill. How much change should you get back?

_____ - _____ = _____

D



E Who Am I?

- I am odd.
- My digits add to 5.
- My digits multiply to 6.

t u

F Who Am I?

- I am a perfect square.
- My units digit is five more than my tens digit.
- I am greater than 20.

t u

Label the two numbers on the number line and use it to find the distance.

G What is the distance between 8 and $2\frac{2}{3}$?

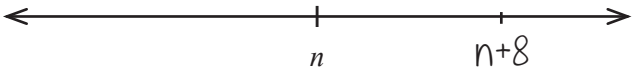
H What is the distance between $3\frac{4}{5}$ and $7\frac{2}{5}$?

I What's the distance between $-1\frac{2}{3}$ and $-12\frac{1}{3}$?

3-5 Picturing Subtraction

① On this number line, n is labeled. Plot:

$n + 8$ $\quad n + -8$ $\quad n - 8$ $\quad n - -8$

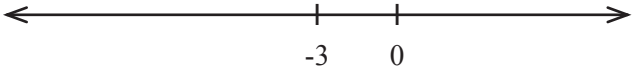


② How do you know where $n - 8$ goes?
How do you know where $n - -8$ goes?

Write your explanations, then share them with a friend.

③ On this number line, plot:

$-3 + 8$ $\quad -3 + -8$ $\quad -3 - 8$ $\quad -3 - -8$



Illustrate each problem on a number line. Then say *only* whether the answer will be **positive** or **negative**.

ex $20 - 26$ is Negative

Do you pass 0?

④ $-3 - 5$ is _____

Where is 0?

⑤ $4.5 - -18$ is _____

Mark the 0

⑥ $102 - 101.9$ is _____

⑦ $-9.1 - -3.64$ is _____

⑧ $-3\frac{3}{8} - -7\frac{7}{8}$ is _____

⑨ Answer **True** or **False** for each statement below. If the statement is false, provide a specific example using numbers or a picture that shows why.

$10b$ is always positive. _____

$-3p$ is always negative. _____

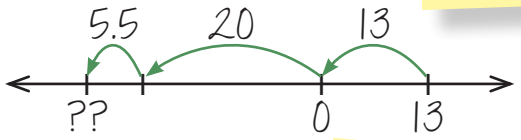
If x is a negative number,
then $x - 1$ is always negative. _____

If h is a positive number,
then $h - 100$ is always negative. _____

Illustrate each subtraction problem on the number line. This time write the answer, too. Keep marking 0 and make as many jumps as you need to help you with your calculation.

10 13 – 38.5 = _____

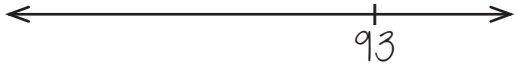
Is this positive or negative?



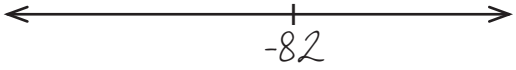
There's no "correct" or "better" way to draw the arrows. You're moving left 38.5, and you can break that up into as many (or as few) pieces as you need to.

Don't forget to keep track of 0

11 93 – 29 = _____



12 -82 – 38 = _____



13 11 – 53 = _____

14 -8 – -5.6 = _____

15 Who Am I?

- My units and my hundreds digit are the same.
- The sum of my units digit and hundreds digit is 10.
- My tens digit is one more than my units digit.

h	t	u

16 Who Am I?

- I am a multiple of 10.
- My tens digit is twice my hundreds digit.
- My hundreds digit is odd.
- My hundreds digit is greater than 1.

h	t	u

17 -4.2 – 36 = _____

18 26 – 8.8 = _____

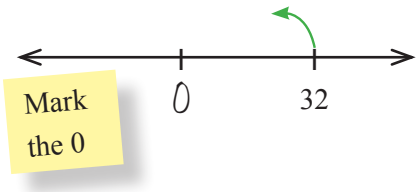
19 539 – 52.1 = _____

20 $-2\frac{1}{3} - 70\frac{1}{3}$ = _____

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.
Illustrate each problem on a number line. Then say *only* whether the answer will be **positive** or **negative**.

Ⓐ 32 – 49 is _____



Ⓑ -8.3 –6.33 is _____

Ⓒ 2 – -2.5 is _____

Ⓓ 7.12 –7.6 is _____

Ⓔ $11\frac{7}{8} - 11\frac{1}{2}$ is _____

Ⓕ -49.62 – -49.7 is _____

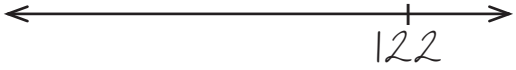
Ⓖ Answer **True** or **False** for each statement below. If the statement is false, provide a specific example using numbers or a picture that shows why.

-4*k* is always negative. _____

If *a* is a negative number, then *a* + 20 is always positive. _____

Illustrate and answer each subtraction problem using the number line.

Ⓗ 122 – 36 = _____



Ⓘ -5.3 – 17.1 = _____

Ⓙ 12 – 20.7 = _____

Ⓚ $30\frac{4}{5} - 24\frac{1}{5}$ = _____

Ⓛ Who Am I?

- The sum of my digits is 16.
- My tens digit is greater than my units digit.

t	u

Ⓜ Who Am I?

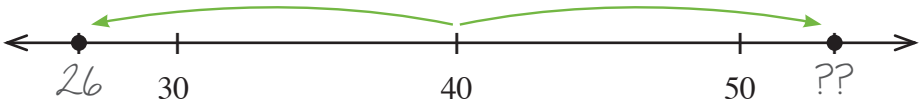
- All of my digits are odd.
- My units digit is a perfect square.
- None of my digits are 1.
- Multiplying *h* and *t* gives *u*.

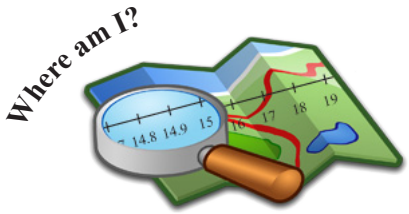
h	t	u

3-6 It's All Related!

Where am I? Mark all possible solutions and fill in all of the blanks.

① Clue: I am 14 units away from 40

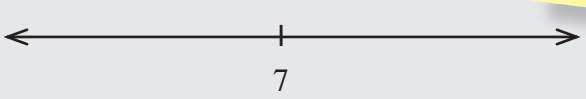




I am x. Where am I? I could be 26 or _____ .

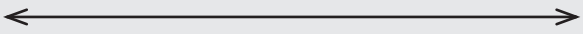
② Clue: The distance between me and 7 is 30

Mark 0



I am x. Where am I? I could be _____ or _____ .

③ Clue: I am 83 units away from 2



I am x. Where am I? I could be _____ or _____ .

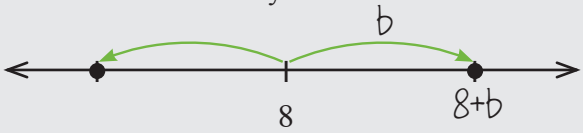
④ Clue: I am 30.6 units away from -5

Where am I? I could be _____ or _____ .

⑤ Clue: The distance between me and -12 is $5\frac{1}{3}$

I am x. Where am I? I could be _____ or _____ .

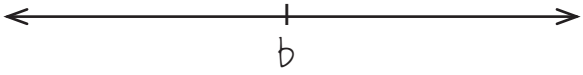
⑥ Clue: I am b units away from 8



I am x. Where am I? I could be 8 + b or _____ .

⑦ For problem 6, if you are told that $8 - b$ is negative, what does that tell you about what numbers b can be?

⑧ Clue: I am 9 units away from b



I am x. Where am I? I could be _____ or _____ .

In these problems, there are two variables. The variable b stands for the distance you move or the number where you start. The variable x stands for the numbers where you land. We say we "write x in terms of b" because where you land (x) depends on our value for b.

⑨ Who Am I?

- All of my digits are different and odd.
- My hundreds digit is my largest digit, and my units digit is my smallest digit.
- $u > 1$
- $h < 9$

h	t	u

⑩ Who Am I?

- I am even.
- My tens digit is a perfect square.
- $u = h$
- I am less than 400.
- My tens digit is greater than 4.

h	t	u

11 Clue: I am 38 units away from c

I am x. Where am I? I could be _____ or _____ .

12 For problem 11, if you are told that $c + 38$ is negative, what does that tell you about what numbers c can be?

13 Answer **True** or **False** for each statement below. If the statement is false, provide a specific example using numbers or a picture that shows why.

Your example can be a picture.

If c is a negative number, then $c + 38$ is always positive. _____

If a is a positive number, then $a - 300$ is always negative. _____

14 Who Am I?

- I am even, and all my digits are even.
- My hundreds digit is four times my units digit.
- My three digits all add up to 10.

h	t	u

15 Who Am I?

- I am odd.
- My hundreds digit is 2.
- My units digit is greater than my tens digit.
- My tens digit is twice my hundreds digit.
- My units digit is a perfect square.

h	t	u

We've seen numbers play two different roles on the number line. They can be **addresses** ("I am 28") or they can be **distances** ("Jump 35 to the right").

We've also seen two ways to think about subtraction. For example, in a problem like 83 – 80:

- The two numbers (83 and 80) can both be thought of as addresses, and the answer is the distance between them (with the correct sign). Or...
- The first number (83) can be thought of as an address and the second number (80) is a distance to jump to the left. Then the answer is the address where you land.

No matter which way you think about subtraction, use the number line as your guide.

16 $73 - 49 =$ _____

17 $15 - 52.8 =$ _____

18 $-3\frac{2}{7} - -14\frac{3}{7} =$ _____

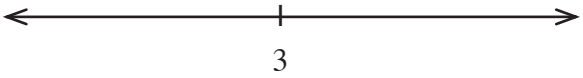
19 $31\frac{1}{5} - 8\frac{3}{5} =$ _____

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

Where am I? Mark all possible solutions and fill in all of the blanks.

A Clue: The distance between me and 3 is 43

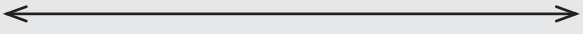


I am *x*. Where am I? I could be _____ or _____ .

B Clue: I am 3.8 units away from -24

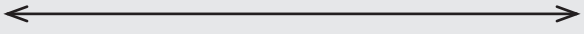
I am *x*. Where am I? I could be _____ or _____ .

C Clue: I am 108 units away from -1.1



I am *x*. Where am I? I could be _____ or _____ .

D Clue: The distance between me and 98 is $15\frac{4}{5}$



I am *x*. Where am I? I could be _____ or _____ .

E Clue: I am 33 units away from *m*

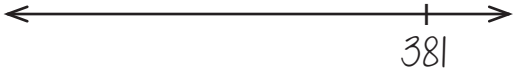
I am *x*. Where am I? I could be _____ or _____ .

F Clue: I am *n* units away from 3.03

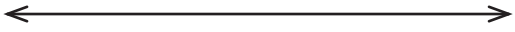
I'm *x*. Where am I? I could be _____ or _____.

Illustrate and answer each subtraction problem using the number line.

G 381 – 136 = _____



H 16.1 – 30 = _____



I $49 - 11\frac{2}{3}$ = _____

J -92 – -11.4 = _____

K Who Am I?

- I am a multiple of 10.
- My hundreds digit is one more than my tens digit.
- My three digits all add up to 7.

h	t	u

L Who Am I?

- I am even.
- My tens digit and units digit are the same.
- My hundreds digit and units digit add up to 17.

h	t	u

Unit Additional Practice Problems

Use these questions to help you prepare for the Unit Assessment.

Find the distances. Sketch a number line in your head or on the paper.

1. What is the distance between 30 and 150? _____

2. What's the distance between -55 and 40? _____

3. What's the distance between 56 and -31? _____

4. What about between -3 and -8.5? _____

5. What's the distance between 7.8 and 9.1? _____

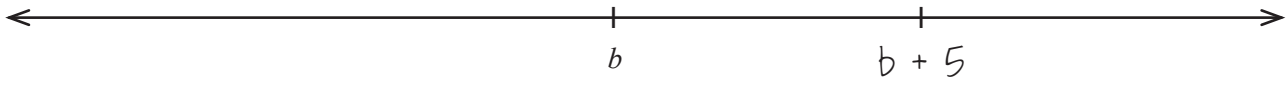
6. How far apart are $10\frac{5}{7}$ and 11? _____

7. How far apart are $10\frac{5}{7}$ and $11\frac{1}{7}$? _____

8. What's the distance between $9\frac{1}{7}$ and $10\frac{5}{7}$? _____

9. Plot the numbers. Be sure to label them. Some of them end up in the same place.

$b + 5$ $b - 2$ $3 + b$ $b - 8$ $b + 3$ $b + -8$ $-3 + b$ $b - -10$



10. Who Am I?
    - I am even.
    - The sum of my digits is 11.
    - My tens digit is a perfect square.

    t u

11. Who Am I?
    - I am odd.
    - My tens digit is 6 more than my units digit.
    - I am less than 90.

    t u

12. Answer **True** or **False** for each statement below. If the statement is false, provide a specific example using numbers or a picture that shows why.

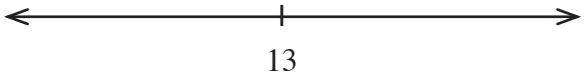
    $-y$ is always negative. _____

    If n is a negative number, then $n + 99$ is always positive. _____

    If g is a positive number, then $g + 13$ is always positive. _____

    If r is a positive number, then $r - 200$ is always negative. _____

13 Clue: The distance between me and 13 is 70



I am *x*. Where am I? I could be _____ or _____ .

14 Clue: I am 8.4 units away from -60

I am *x*. Where am I? I could be _____ or _____ .

15 A sandwich costs $6.77. You pay with a $20 bill. How much change should you get back?

_____ - _____ = _____

Illustrate each problem on a number line. Then say *only* whether the answer will be **positive** or **negative**.

16 26 – -39 is _____

17 -4 – 5 is _____

18 43.2 – 43.187 is _____

Illustrate and solve.

19 24 – 40 = _____

20 -18 – 17.2 = _____

21 -56 – -3.9 = _____

22 $17\frac{1}{5} - 4\frac{3}{5}$ = _____

23 Who Am I?

- All of my digits are different.
- None of my digits is 0.
- All of my digits are perfect squares.
- My units digit is my only even digit.
- I am less than 500.

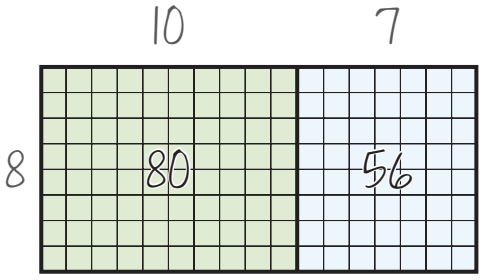
h	t	u

24 Who Am I?

- I am even.
- My tens digit is 1.
- My three digits multiply to 12.
- My hundreds digit is four less than my units digit.

h	t	u

Unit 4: Area and Multiplication



$$8 \cdot 17 = 8 \cdot 10 + 8 \cdot 7 = 80 + 56 = 136$$

	x	$2y$	-5
$4x$	$4x^2$	$8xy$	$-20x$
-3	$-3x$	$-6y$	15

$$(x + 2y - 5)(4x - 3) = 4x^2 + 8xy - 23x - 6y + 15$$

Transition to Algebra

Unit 4: Area and Multiplication

Lessons in this Unit:

1: Puzzles and Patterns
2: Multiplication Patterns
3: Measuring Area
4: Area Models and Equivalent Expressions
5: Modeling Area with Algebra
6: Squaring Variables
7: Equivalent Algebraic Expressions
8: Extending the Distributive Property

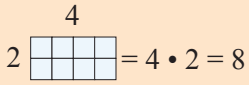
You will learn to solve puzzles like this...

MysteryGrid 1-2-3-4 Puzzle

8, +	2, ÷		3
	4, ÷		4, x
	6, x		
		7, +	

Algebraic Habits of Mind: Using Structure

We can see how the area of a small (like 2 by 4) rectangle is given by multiplying base and height:



$= 4 \cdot 2 = 8$

We also know that if we divide up the rectangle into smaller pieces, we still get the same overall area:

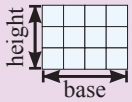
We know this because we understand the underlying **structure** of area. We know how area works because we can see it with our eyes. In this unit, we'll find out how to use this structure to solve problems we can't see so easily, like $562 \cdot 87$ or $(x + 2)(x + y - 3)$.

What is Area?

Area is the amount of 2-dimensional "stuff inside" a shape. It's not actually a number, but we can use numbers to keep track of it. We measure the area of a shape by comparing it to a little chunk of area, one "unit". Sometimes □ = 1 unit; sometimes a square foot = 1 unit; sometimes an acre = 1 unit. Different scales each use a different unit to measure area.

If the shape is a rectangle and □ = 1 unit, we can find the area by multiplying the number of rows times the number of columns. The area of  $= 2 \cdot 4 = 8$

Because all the rows of a rectangle stack up with the same number of units, the area always works the same way for every rectangle. Area = base x height.



We can also measure area even when we don't know the measurements. For example, if we know that a room is x feet long and y feet wide, then we know that the area of the floor is x times y, or just xy, even before we do the measuring because it works for every rectangle. This is why we can use *area models* to help multiply algebraic expressions even when we don't know what the variables equal.

4-1 Puzzles and Patterns

① MysteryGrid **1-2-3** Puzzle

2, –	2, ÷	1 2
	12, x	3, ÷

② MysteryGrid **3-4-5** Puzzle

2, –		7, +
20, x	4	
	8, +	

③ MysteryGrid **6-7-8** Puzzle

23, +	42, x	
		48, x
13, +		

MysteryGrid Puzzle Instructions

You are allowed to write only the numbers that appear in the puzzle's name.

Each row and column must contain exactly one of each number.

Each "room" (space with a heavy border) contains a "target number" and often an operation. The numbers you write in the room must make the target number using the operation. For example:

- 3, – means make 3 using subtraction.
- 7, + means make 7 using addition.

Order doesn't matter. To fill in a room like this:

2, –	

in a MysteryGrid 2-3-5 Puzzle, the only numbers that can work are 3 and 5, which you can draw either way:

2, – 3	5	or	2, – 5	3

If you don't know yet which order is correct, don't guess! (It could mess you up later!) Just write what you *do* know like this:

2, – 3 5	

You might want to use pencil so you can erase if you need to.

Other clues will help you sort out the order later.

④ MysteryGrid **1-2-3** Puzzle

4, x	9, x	
6, +		

When you have a puzzle, ask yourself, "what is the easiest place to start?"

⑤ MysteryGrid **3-4-5** Puzzle

15, x 3 5	11, +	
		8, +
20, x 4		

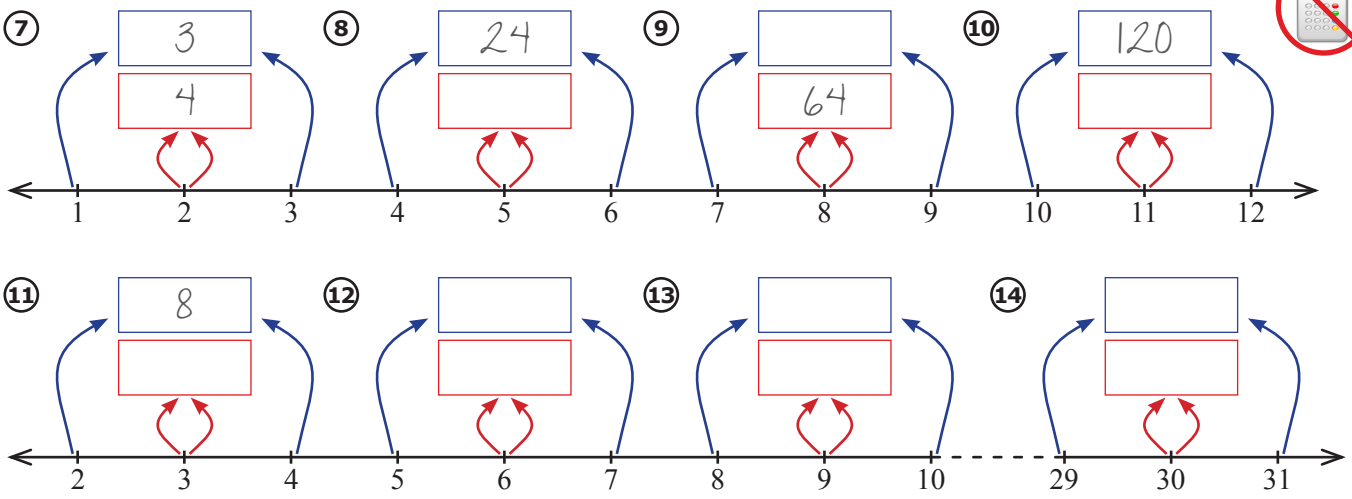
⑥ MysteryGrid **2-3-5** Puzzle

30, x	45, x	
		20, x

Algebraic Habits of Mind: Puzzling and Persevering

Good problem solvers look for the easiest place to start, whether they are doing a puzzle or a test. They also keep track of what they know and choose their next move based on the discoveries they've already made.

In each problem, multiply the middle number by itself and the two outside numbers together.



Some patterns are very easy to describe with words. For example: "start with 1 and add 2 again and again" (you get: 1, 3, 5, 7, 9, and so on). Other patterns can be much harder to describe with words, as Michael is about to discover. Algebra provides a language that makes it easier to describe mathematical patterns.

Thinking out Loud

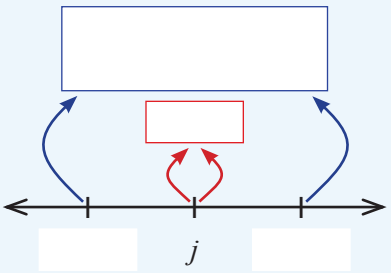
Michael: So, when you take one number and you multiply it by itself and then subtract one from that, you get the two numbers on each side multiplied togeth... uh, it's hard to describe with words...

Jay: Let's name the center number j! *(Jay writes j.)*

Lena: Oh, and then we can describe its neighbors as... *(She stops talking and just writes.)*

Pausing to Think

Finish Lena's thought. How might she name the two neighbor numbers?



Michael: Right, much better! So this number *(he points to the red box)* is j times j, and this number *(he points to the blue box)* is...

Lena: Sorry, Michael, I wasn't paying attention...

Michael: *(Grumbles)* So $j - 1$ times $j + 1$ is the same as...

Discuss & Write

Finish Michael's reasoning.

Michael: But which is it? $j - 1$ times $j + 1$ or j times j minus 1?

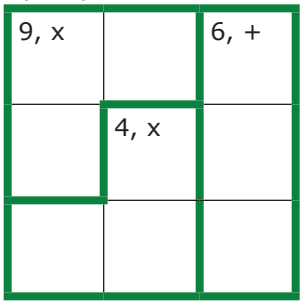
Jay: I think we're going to have to learn how to multiply with algebra to answer that question!

Additional Practice Problems

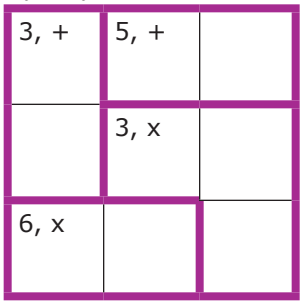
Select problems that will help you learn. Do some problems now. Do some later.

Solve these MysteryGrid puzzles using the rules on page 2.

A MysteryGrid 1-2-3 Puzzle



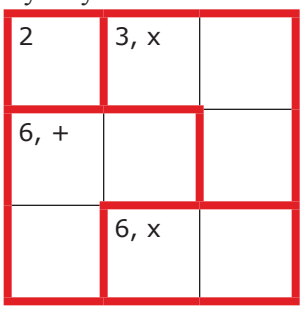
B MysteryGrid 1-2-3 Puzzle



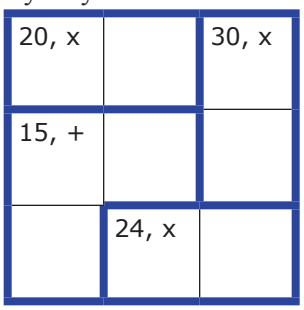
Don't write the numbers in until you are absolutely sure. Use little notes instead.

Like this: 6, x (3, 2)

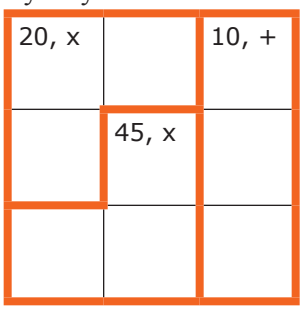
C MysteryGrid 1-2-3 Puzzle



D MysteryGrid 4-5-6 Puzzle

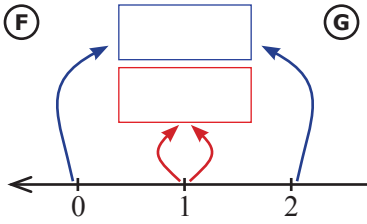


E MysteryGrid 2-3-5 Puzzle

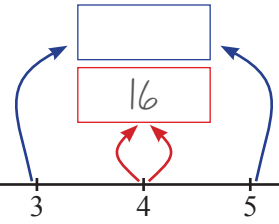


In each problem, multiply the middle number by itself and the two outside numbers together.

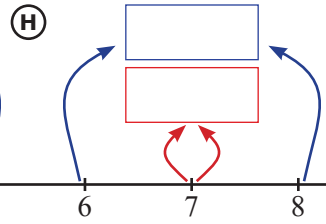
F



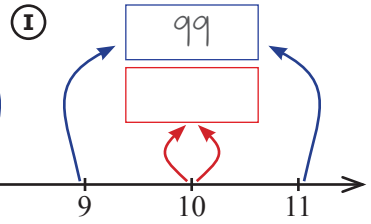
G



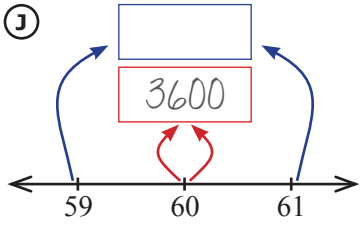
H



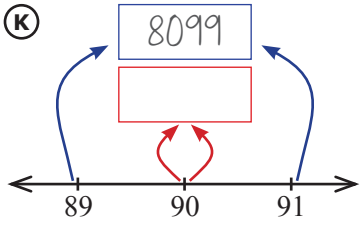
I



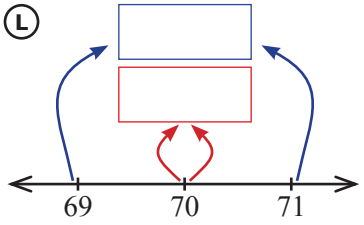
J



K



L



M Who Am I?

- I am even.
- I am between 35 and 55.
- I am a multiple of 8.
- I am not a multiple of 5.

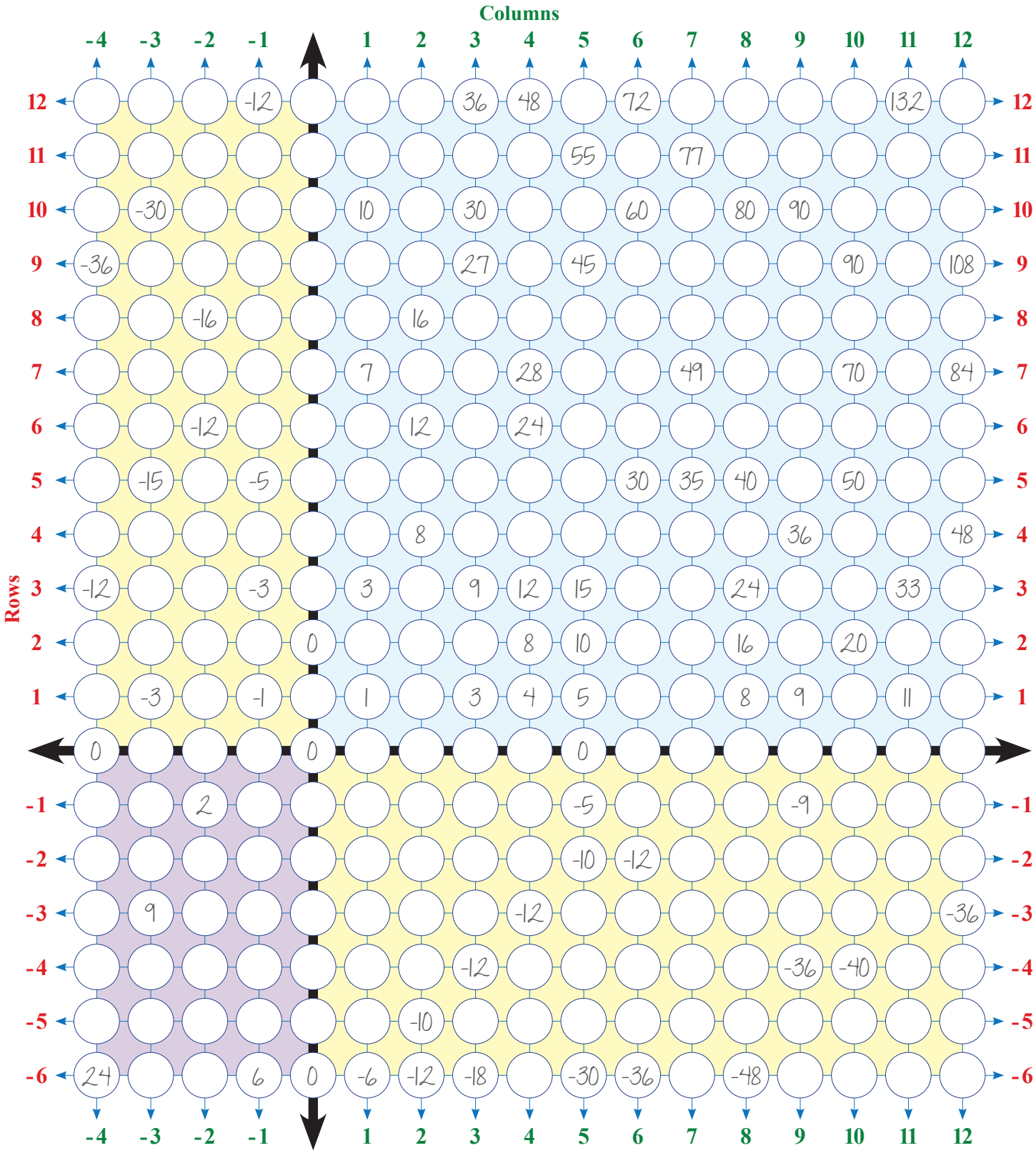
t u

N Who Am I?

- Both of my digits are odd.
- My digits are not the same.
- I am divisible by 3.
- I am one less than a perfect square.

t u

4-2 Multiplication Patterns



What *number* is located at...

① **column 2**, **row 12**? ② **column -3**, **row 8**? ③ **column -4**, **row -6**? ④ **column 12**, **row -2**?

⑤ List several locations of 12.

Column	6											
Row	2											

Discuss & Write How would you describe the location of the zeros in the chart?

Determine whether the number at the following locations is *negative or positive*.

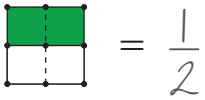
⑥ **column 5**, **row -4** ⑦ **column 8**, **row 4** ⑧ **column -2**, **row 10** ⑨ **column -5**, **row -6**

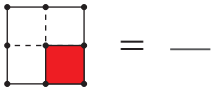
Discuss & Write What You Know

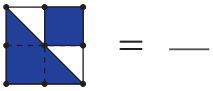
Where are the negative numbers in the chart? What is one way to explain why they show up there?

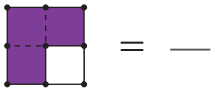
⑩ If we agree that... = 1

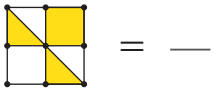
Figure out the area of each of these shaded sections.

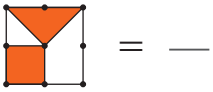
 = $\frac{1}{2}$

 = —

 = —

 = —

 = —

 = —

These Latin Squares are like MysteryGrid puzzles without all the rules. Just follow the instructions beside them.

⑪ 1-2-3 Latin Square

3	2	
	1	

Fill in the grid with exactly one 1, one 2, and one 3 in every column and row.

⑫ *x-y-z* Latin Square

		z
	y	

Use x, y, and z exactly once in every column and once in every row.

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

What *number* is located at...

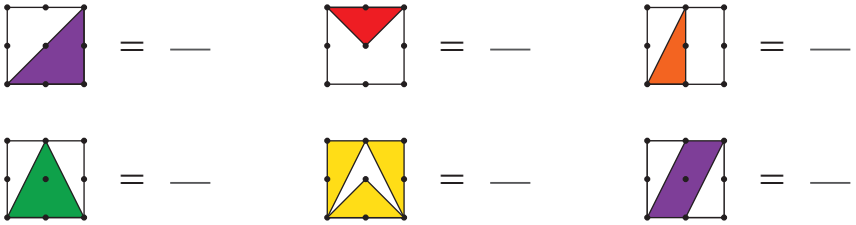
Ⓐ **column 4**, **row 7**? Ⓑ **column -4**, **row -4**? Ⓒ **column 3**, **row -5**? Ⓓ **column -1**, **row 11**?

Ⓔ List several locations of 24.

Column	-4											
Row	-6											

Ⓕ If we agree that... = 1

Figure out the area of each of these shaded sections.



These Latin Squares are like MysteryGrid puzzles without all the rules. Just follow the instructions beside them.

Ⓖ 4-5-6 Latin Square

4		
	5	

Fill in the grid with exactly one 4, one 5, and one 6 in every column and row.

Ⓗ *r*-*s*-*t* Latin Square

	s	
		r

Use *r*, *s*, and *t* exactly once in every column and once in every row.

Solve these MysteryGrid puzzles using the rules on page 2.

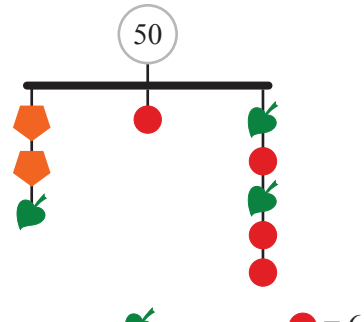
Ⓘ MysteryGrid **1-2-3** Puzzle

6, x		
9, x		3, +
2		

Ⓙ MysteryGrid **5-7-9** Puzzle

35, x		63, x
45, x		
7	14, +	

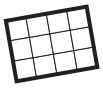
Ⓚ

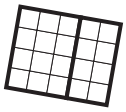


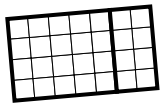
4-3 Measuring Area

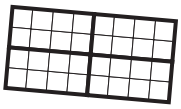
Let □ = 1. Determine the area of each figure.

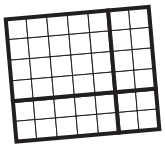
① 

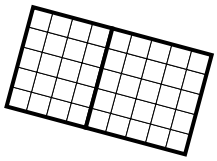
② 

③ 

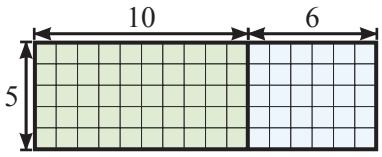
④ 

⑤ 

⑥ 

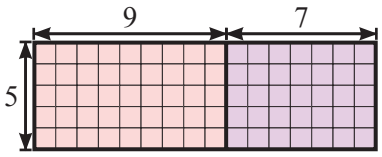
⑦ 

⑧ The same 5 by 16 rectangle is cut in three different ways. For each one, find the area of each part and the total area.

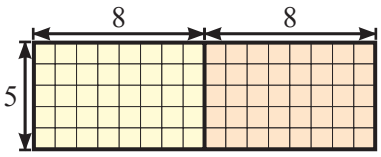


$5 \cdot 16 = 5 \cdot 10 + 5 \cdot 6 = \underline{\quad} + \underline{\quad} = \underline{\quad}$ total area

Area models



$5 \cdot 16 = 5 \cdot 9 + 5 \cdot 7 = \underline{\quad} + \underline{\quad} = \underline{\quad}$ total area



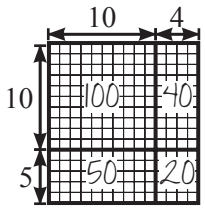
$5 \cdot 16 = 5 \cdot 8 + 5 \cdot 8 = \underline{\quad} + \underline{\quad} = \underline{\quad}$ total area

Which rectangle in problem 8 was easiest to calculate? Why?

Discuss & Write What You Think

Explain why these three rectangles all have the same total area.

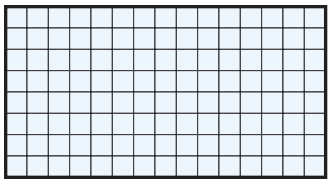
⑨ Jing drew this sketch to think about the problem 15 • 14 and started solving it. What should she do next?



⑩ 5-7-9 Latin Square

7		
		9

⑪ Determine the area of this figure by finding a *convenient* way to cut it up.

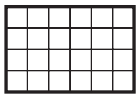


Discuss & Write

What made your choice feel convenient?
How did you make your decision about how to cut?

Determine both the area and perimeter of each figure.

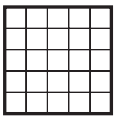
⑫



Area: _____

Perimeter: _____

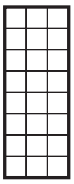
⑬



Area: _____

Perimeter: _____

⑭



Area: _____

Perimeter: _____

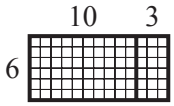
The **perimeter** is the distance all the way around the edge of a figure.
The perimeter of □□ is 6.
But the area is, of course, 2.

Discuss & Write What You Think

Describe the difference between area and perimeter.

Use the area models to solve each problem.

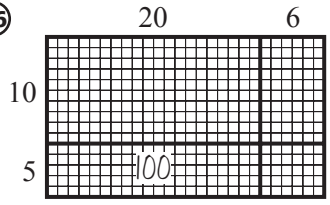
⑮



6 • 13 = ____ + ____ = _____

Area models

⑯



15 • 26 = _____

Discuss & Write What You Think

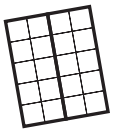
Kayla says that breaking up the area, calculating each part, and adding them up reminds her of finding distance on the number line because you have to break up the distance with jumps, calculate each part, and add them up. What do you think of that?

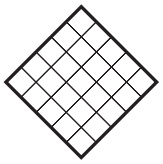
Additional Practice Problems

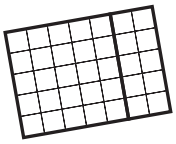
Select problems that will help you learn. Do some problems now. Do some later.

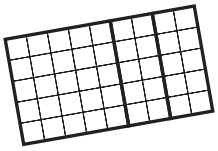
Let □ = 1. Determine the area of each figure.

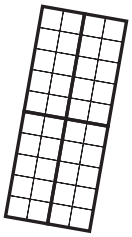
(A) 

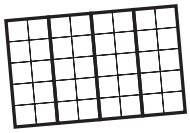
(B) 

(C) 

(D) 

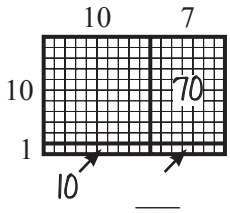
(E) 

(F) 

(G) 

Use the area models to solve each problem.

(H)

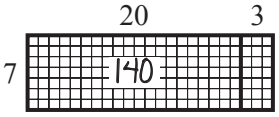


11 • 17

= ____ + ____ + ____ + ____

= _____

(I)



7 • 23

= ____ + ____

= _____

Solve this MysteryGrid puzzle using the rules on page 2.

(J) Who Am I?

- I am a multiple of 5.
- I am between 72 and 92.
- My tens digit is even.
- I am not divisible by 4.

t u

(K) MysteryGrid **2-3-4** Puzzle

9, +	12, x	
	12, x	
	4	

(L) If we agree that...

 = 1

Figure out the area of each of these shaded sections.

 = ___

 = ___

 = ___

 = ___

 = ___

 = ___

4-4 Area Models and Equivalent Expressions

Fill in all the blanks and use the area models to solve each problem.

①

	20	5
10		
2		

25 • 12 = _____

②

	60	4
10		
5		

15 • 64 = _____

③

	50	1
20		
4		

24 • 51 = _____

④

	40	1
500		
60	2400	
2		

562 • ____ = _____

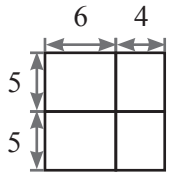
Draw an area model and use it to solve these problems.

⑤ 23 • 46 = _____

⑥ 35 • 154 = _____

⑦ 15 • 43 = _____

⑧ Which of the following expressions corresponds to the diagram on the left? Circle all that apply.



Ⓐ (5 + 5)(6 + 4)

Ⓑ 30 + 20 + 30 + 20

Ⓒ (6 + 5)(4 + 5)

⑨ Fill in this grid with exactly one 1, one 2, and one 3 in every column and row.

1-2-3 Latin Square

		2
3		

⑩ What do these expressions all have in common? Circle all that apply.

(5 + 5) • (6 + 4)
$(6 + 4)^2$
10 • (5 + 5)

Ⓐ They all have the same final answer.

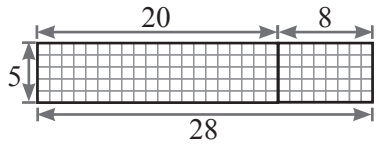
Ⓑ They are just different ways of writing 10 • 10.

Ⓒ They have nothing in common.

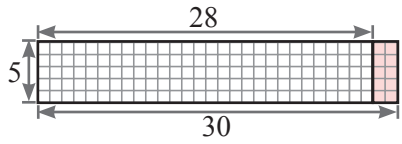
Discussing Together

Discuss and agree on your answer(s) to problem 9.
Draw models to explain your reasoning.

11

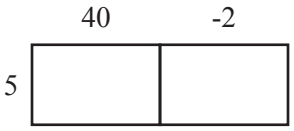


$5 \cdot 28 = 5(\_\_\_\_ + 8) =$ _____



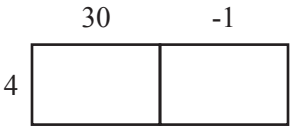
$5 \cdot 28 = 5(30 - \_\_\_\_) =$ _____

12



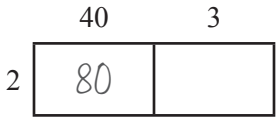
$5 \cdot 38 =$ _____

13



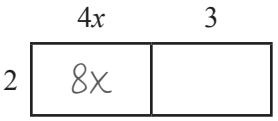
$4 \cdot 29 =$ _____

14



$2 \cdot (40 + 3) =$ _____

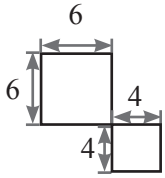
15



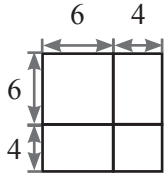
$2 \cdot (4x + 3) =$ _____

Attach the matching expression to each diagram.

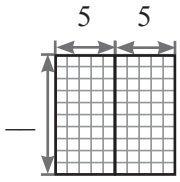
16



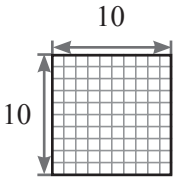
17



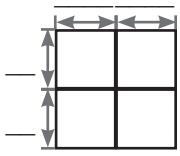
18



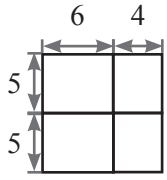
19



20



21



Complete each grid with exactly one 1, 2, 3, and 4 in every column and row.

22 **1-2-3-4** Latin Square

	3	1	
			4
3			
	4	2	

Look for the easiest place to start and only write things in once you are sure!

23 **1-2-3-4** Latin Square

3			4
			2
	1		
		2	

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

Fill in all the blanks and use the area models to solve each problem.

48x52 and 52x48 are the area of the same rectangle, so the answers will be the same.

A

	70	4
20	1400	
5		

25 • 74 = _____

B

	50	2
40		
8		

52 • 48 = _____

C

	20	1
60		
9		

69 • 21 = ____ + ____ + ____ + ____ = _____

D

	100	50	2
40			
8	800		

152 • 48 = _____

Use area models to solve these problems.

E 45 • 21 = _____

	____	____
____		
____		

F 64 • 312 = _____

	____	____	____
____			
____			

G 53 • 24 = _____

H Draw an area model for the expression

$10 \cdot (3 + 7)$

I 2-3-4 Latin Square

	2	
4		

J If we agree that...

 = 1

Figure out the area of each of these shaded sections.

 = ___

 = ___

 = ___

 = ___

 = ___

 = ___

4-5 Modeling Area with Algebra

① MysteryGrid **1-2-3-4** Puzzle

1, −	1, −		3, −
	2, ÷	2	
3, −		7, +	
	6, x		

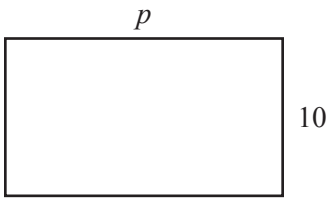
② MysteryGrid **1-2-3-4** Puzzle

4	2, ÷		1, −
6, +		12, x	
1, −			
	5, +		2

③ MysteryGrid **1-2-3-4** Puzzle

48, x		6, x	
		3, ÷	2, ÷
3, +	2		
	7, +		1

④ Imani wants to build puppy pen. If her pen is 10 feet long in one direction and p feet long in the other direction, how much total room will the puppies have to play? Explain your reasoning.



⑤

3	20	5
	60	

$3 \cdot (20 + 5) =$ _____

⑥

3	2b	5
	6b	

$3 \cdot (2b + 5) =$ _____

Thinking out Loud

Michael: Ok, I get that problem 5 is a model of a 3 by 25 rectangle and we are splitting it up to find the area easily, but what is going on in problem 6? What does a 3 by $2b + 5$ rectangle even mean?

Lena: Let's think back to the puppy pen problem. We didn't know one of the dimensions of the pen, so we said it was p. So, p times the 10 foot side we know makes $10p$.

Jay: That's $10p$ square feet. Not just $10p$. The dimensions are in feet so the area is in square feet.

Lena: Ok fine. $10p$ square feet.

Michael: I can see how problem 6 is like that. The 3 is the distance we know for sure and the $2b$ is two times some distance we don't know. When we multiply them together we get area: $6b$ square... Square what? There are no units! There are no feet or anything!

Lena: Yup, so it's just 6b in that box, but we have to remember that we're multiplying the 3 not just by 2b, but by 2b + 5. Right?

Michael: Right! So, 3 times $2b$ is $6b$ and 3 times 5 is 15, so it's $6b + 15$.

These rectangles are made of smaller rectangles. The labels give the dimensions of the *small* rectangles. Fill in the blanks. Then find the *base, height, and area* of the *large* rectangles. (The drawings are not to scale, but the dimensions are correct.

7

	8
x	$8x$
3	24

Area = $8(x + 3)$ (base x height)

Area = _____ (total)

8

	7
$2x$	$14x$
3	

Area = $7(\quad + \quad)$ (base x height)

Area = $14x +$ _____ (total)

Fill in the missing pieces in these multiplication problems.

9

	b	____
-7		

$-7(b + 8) =$ _____

10

	____	____
____		

$-4(5b + 8) =$ _____

11

	y	-1
8		

$8(y - 1) =$ _____

12

	$3k$	4
7		

$7(3k + 4) =$ _____

13

	$4c$	-3
b		
-8	$-32c$	

$(b - 8)(4c - 3) =$ _____

14

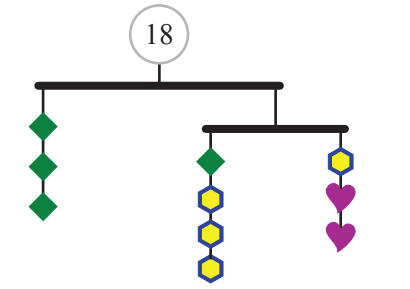
	____	____
____		
____		

$(2b - 8)(4c - 3) =$ _____

15 MysteryGrid 1-2-3-4 Puzzle

8, x			5, +
8, x	9, x		
	8, +		1
		2, –	

16



◆ = _____ ⬢ = _____ ♥ = _____

17 Who Am I?

- I am even.
- I am a multiple of 9.
- I am less than 70.
- $t > u$

t u

18 Who Am I?

- I am a multiple of 7.
- My tens digit is less than my units digit.
- Both of my digits are even.

t u

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

Fill these in completely.

A

	100	1
20		
3		3

23 • 101 = ____ + ____ + ____ + ____ = _____

B

	100	10	1
20	2000		
3			

23 • 111 = _____

C

	50	4
10	500	

10 • 54 = _____

D

	40	-1
5		

5 • 39 = _____

Use an area model to multiply these algebraic expressions.

E

	x	4
10	$10x$	

$10(x +$ ____ $) =$ _____

F

	$5y$	-4
2		

$2($ _____ $) =$ _____

G

____	____	$-k$
____		

$y(8 - k) =$ _____

H

	k	____
____		

$y(k - 8) =$ _____

I

	x	7
y		
-5		

$(y - 5)(x + 7) =$ _____

J

	____	____
b		
8		-24

$(2a - 3)(b + 8) =$ _____

Use an area model (on paper or in your head) to solve these two problems.

K 2 • 32 = _____

L 3 • 51 = _____

M Who Am I?

- I am a multiple of 6.
- $t = 2u$
- I am between 5^2 and 8^2, but I am not a perfect square.

t	u

N MysteryGrid 1-2-3-4 Puzzle

8, x	3	6, x	3, –
	4, x		
3			1, –
8, x			

4-6 Squaring Variables

Thinking out Loud

①

	x	2
x		
3		

$(x + 3)(x + 2) =$

Michael: Hmm, I know what to do in most of these boxes, but how do I write x times x? x "dot" x?

Lena: Yeah, but $x \bullet x$ is x squared, so I just wrote x^2.

Michael: Oh, right. But now that I'm thinking about it, I wonder why it is called "squared. What does "x squared" even mean?

	x
x	x^2

Lena: Hmm. We've been drawing all these rectangles. I have an idea. Instead of x times x, let's imagine 5 times 5 first. If we drew a 5 by 5 rectangle...

	5
5	25

Michael: *(interrupting)* That'd be a square because the sides are the same!

Lena: Right! And how do we get the area?

Michael: 5 times 5 is 25. Oh! That's why it's called "five *squared*!"

Lena: Yup! And so if it was x by x...

Michael: It would be a square that has sides x long and an area of x^2. Nice!

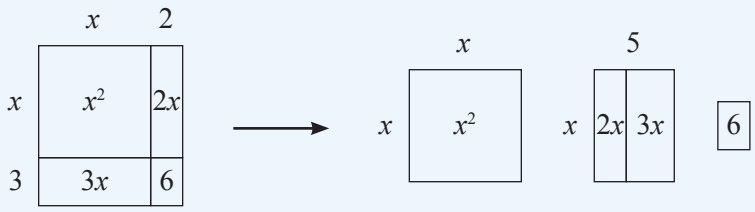
Pausing to Think

What's the area of a 7 by 7 rectangle?
How about a square that measures z on each side?

Lena: Then to finish problem 1, we just add up the partial areas and we have $x^2 + 2x + 3x + 6$.

Michael: And $2x + 3x$ is $5x$, like adding 2 buckets plus 3 buckets. So then we add in the x^2?

Jay: Well, $2x + 3x$ is $5x$, but you can't add in the x^2 the same way. Look at this drawing:



Michael: So, we know that the $2x$ and $3x$ are $5x$ by lining them up and adding them. Then what about x^2?

Jay: x^2 is a *different number* than x. We can't add different numbers if we don't know what they are. We can add $2x + 3x$ like buckets without knowing what's inside; the things we're counting are the same.

Michael: So we get $x^2 + 5x + 6$. Hey, the 5 is from $2 + 3$ and the 6 is from $2 \bullet 3$. Does that always happens?

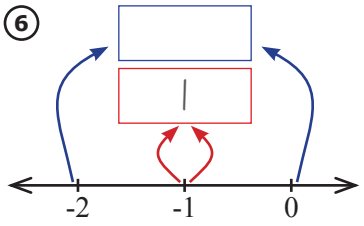
Algebraic Habits of Mind: Seeing and Describing Patterns

Research Michael's last idea. Try different numbers. Does that pattern always happen?

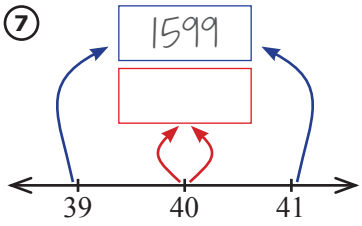
② $b \cdot b =$ _____ ③ $3x \cdot 3x =$ _____ ④ $2y \cdot y =$ _____ ⑤ $-k \cdot -k =$ _____

Multiply the middle number by itself and the two outside numbers together.

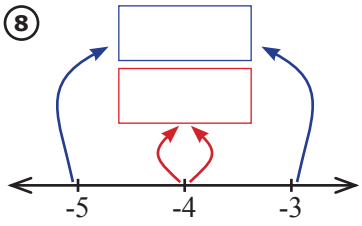
⑥



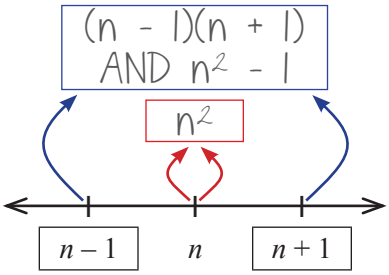
⑦



⑧



Here is the general pattern you discovered for these problems.



Use the area model to prove that both expressions in the blue box are the same.

⑨

	n	1
n		
-1		

$(n-1)(n+1) =$ _____

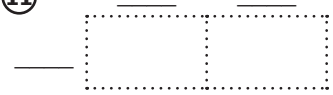
Discuss & Write What You Think

Explain how the area model shows that $(n-1)(n+1)$ is the same as $n^2 - 1$.

Use an area model to multiply these expressions.

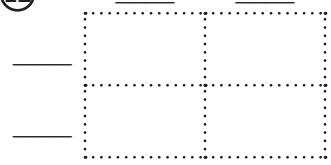
⑩ $b(b+7) =$

⑪



$m(7-m) =$

⑫

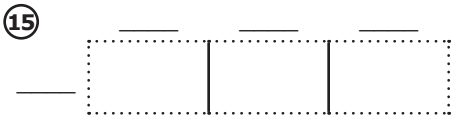


$(b+7)^2 =$

⑬ $(k+5)(k+4) =$

⑭ $x(x+3) =$

⑮



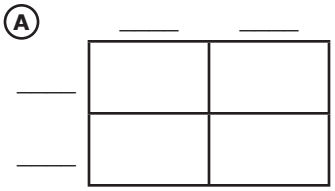
$-x(-x+y+3) =$

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

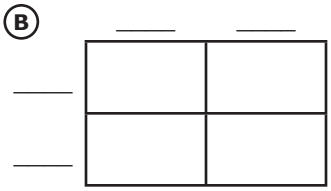
Use an area model to multiply these expressions.

A



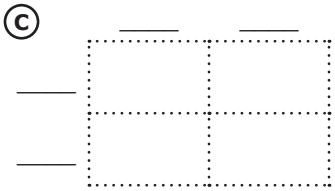
$(12 + s)(s - 4) =$

B



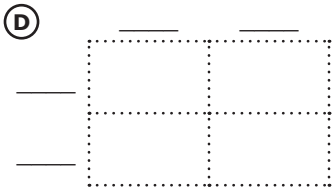
$(m - 6)(m - 5) =$

C



$(x + 3)(x + 7) =$

D



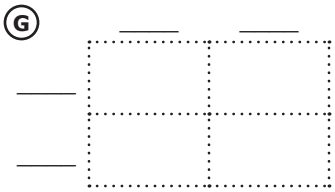
$(x - 8)(x - 5) =$

E $(k + 8)^2 =$

$(k + 8)^2$
is the same as
$(k + 8)(k + 8)$

F $(x + y)^2 =$

G



$(n + 6)(n + 7) =$

H $(4 - p)(p - 4) =$

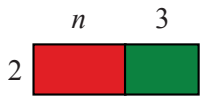
I $(3w + 6)(w - 5) =$

J MysteryGrid 1-2-3-4 Puzzle

6, +	6, x		10, +
		7, +	
4, ÷			
8, x			

4-7 Equivalent Algebraic Expressions

①

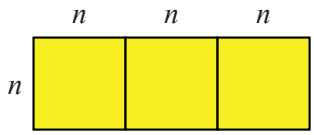


Attach the matching expressions and descriptions here.

②

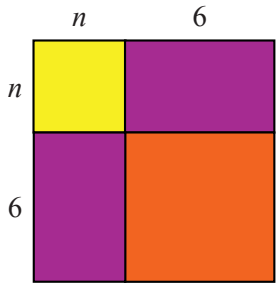
Attach the matching expressions and descriptions here.

③



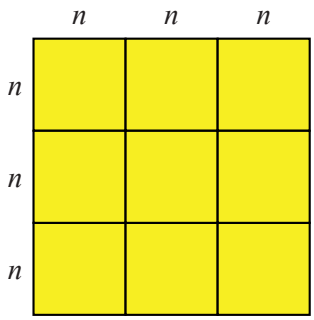
Attach the matching expressions and descriptions here.

④



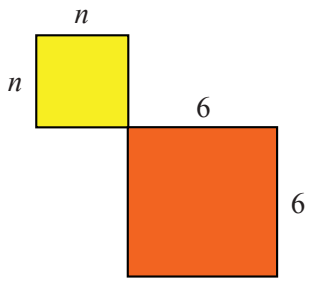
Attach the matching expressions and descriptions here.

⑤



Attach the matching expressions and descriptions here.

⑥

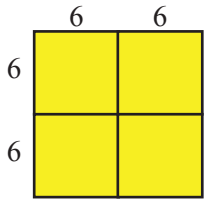


Attach the matching expressions and descriptions here.

Additional Practice Problems

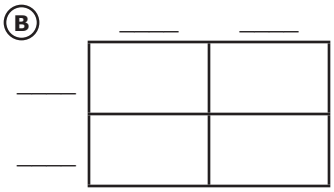
Select problems that will help you learn. Do some problems now. Do some later.

A We know that 12 • 12 is the same as (6 + 6) • (6 + 6). Use the diagram to explain why 12 • 12 is the same as 36 • 4.



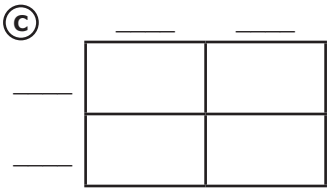
Use an area model to multiply these expressions.

B



$(n + 3)^2 =$

C



$(n + 3)\,(n + 4) =$

D $(x + 5)(x + 2) =$

E $(x + y)^2 =$

F $(5x + 10)(3x + 1) =$

G $(x + 2)(15x + 5) =$

H MysteryGrid 1-2-3-4 Puzzle

8, x		2, –	
	9, x	2	24, x
		16, x	
2			

I Who Am I?

h	t	u

- All of my digits are divisible by 3.
- All of my digits are different.
- None of my digits are less than 2.
- My hundreds digit is greater than my tens digit and less than my units digit.

4-8 Extending the Distributive Property

Jing and Mali both did some parts of these problems wrong. Circle their mistakes, and find the correct answers.

① Jing:

	$3k$	2
$-t$	$-3kt$	$-2t$
$3k$	$6k$	$5k$

$(-t + 3k)(3k + 2) =$ $-3kt - 2t + 11k$

Mali:

	$3k$	2
$-t$	$3kt$	$2t$
$3k$	$9k^2$	$6k$

$(-t + 3k)(3k + 2) =$ $3kt + 2t + 9k^2 + 6k$

What is the correct product? $(-t + 3k)(3k + 2) =$ _____

② Jing: $x(x + 4n + 2) =$ $2x + 4nx + x^2$

Mali: $x(x + 4n + 2) =$ $x^2 + 4n + 2x$

	___	___	___
x			

What is the correct product? $x(x + 4n + 2) =$ _____

Use an area model to multiply.

③ $(x + y + z)(x + 2y) =$ _____

	x	y	z
x			
$2y$			

④ $(m - 8)(2m + n - 9) =$ _____

	___	___	___
___			
___			

Discuss & Write What You Think

Does $2(3n + 3) = 6n + 3$? Show how you know.

Use an area model to multiply these expressions.

⑤

$2(a + 5) =$

⑥

_____ • $(2p + 7) = 8p + 28$

⑦ $(f + 7)(2g + 3) =$

⑧ $(5 + y)(z - 6) =$

⑨ $(c + 6)(2c + 11) =$

⑩ $w(2p + c - 7) =$

⑪ $(c + 6)(2c + 11) =$

⑫ $w(2p + c - 7) =$

⑬

x	x^2	$6x$
5		30

($x + 6$) (____ + ____) = x^2 + _____ + 30

⑭

y	y^2	$2y$
4	$4y$	8

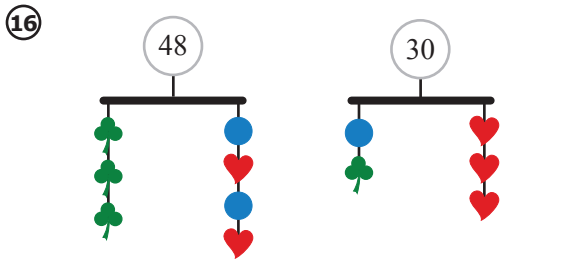
($y + 4$) (____ + ____) = $y^2 + 6y + 8$

⑮ Who Am I?

h	t	u

- I am odd.
- I am a multiple of 5.
- $t > u$
- My tens digit is a perfect square.
- My hundreds digit is three less than my units digit.

⑯



Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

Use an area model and to multiply these expressions.

A $2(y - 19) =$

B $(m - 4)(n + 3) =$

C

	-w	3
____		
-3		

$(-w + 3)(w - 3) =$

D

	x	____	-3
____	$4x^2$		

$4x(x + y - 3) =$

E

	____	4
____	x^2	
8		32

(____ + ____) (____ + ____) = $x^2 + 12x + 32$

F

	____	____
____	x^2	4x
____	3x	12

(____ + ____) (____ + ____) = $x^2 + 7x + 12$

G Who Am I?

- I am odd.
- My tens digit is a perfect square.
- $h < t < u$
- My hundreds digit is one less than my tens digit.
- $h + t = u$

h	t	u

H MysteryGrid 1-2-3-4 Puzzle

4, x	5, +	8, x	
		9, x	
3, +			8, x
12, x			

Use an area model to multiply these expressions.

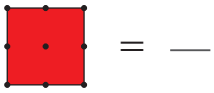
I $(n + 3m)(4n + m) =$

J $5f(-2g + -4h + 9) =$

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

K If we agree that... Figure out the area of each of these shaded sections.

 = ___

 = $\frac{1}{2}$

 = ___

 = ___

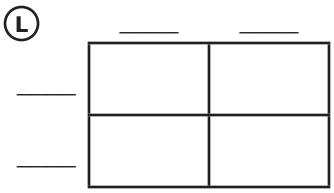
 = ___

 = ___

 = ___

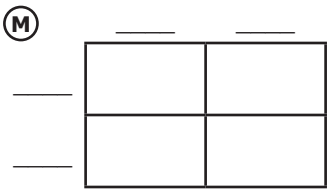
Use an area model to multiply these expressions.

L



$(y + 5)(5 - z) =$

M

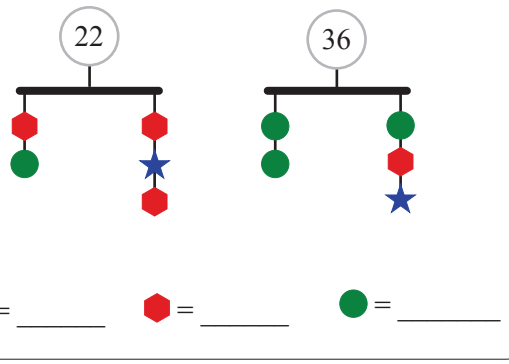


$(y + 5)(z + 5) =$

N $3(4x + 8) =$

O $(b - 2)^2 =$

P



Q MysteryGrid 1-2-3-4 Puzzle

24, x		3, −	
	9, +		
	2, −		6, +
2, ÷			

R Who Am I?

h	t	u

- I am odd.
- My hundreds digit is my only even digit.
- My units digit is my largest digit.
- $u = h + 1$
- $u = 3t$
- The sum of my digits is 20.

Unit Additional Practice Problems

Use this page to prepare for the unit exam.

Use an area model to multiply these expressions.

① $-3(a - 7) =$

② $(a + 2b)(a + b) =$

③ MysteryGrid **1-2-3** Puzzle

6, x		4, +
2, x		
	1, –	

④ **1-2-3-4** Latin Square

	4		3
		2	
	2		
1			

Match the algebraic expression on the left with the area model on the right.

⑤ $2n + 6$

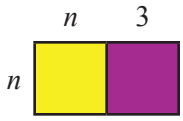
⑥ $5n$

⑦ $2n + 12$

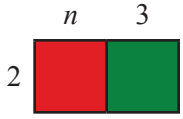
⑧ $n(n + 3)$

⑨ $2n + 3$

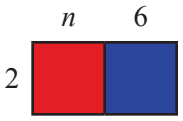
Ⓐ



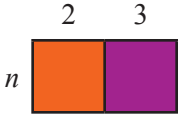
Ⓑ



Ⓒ



Ⓓ



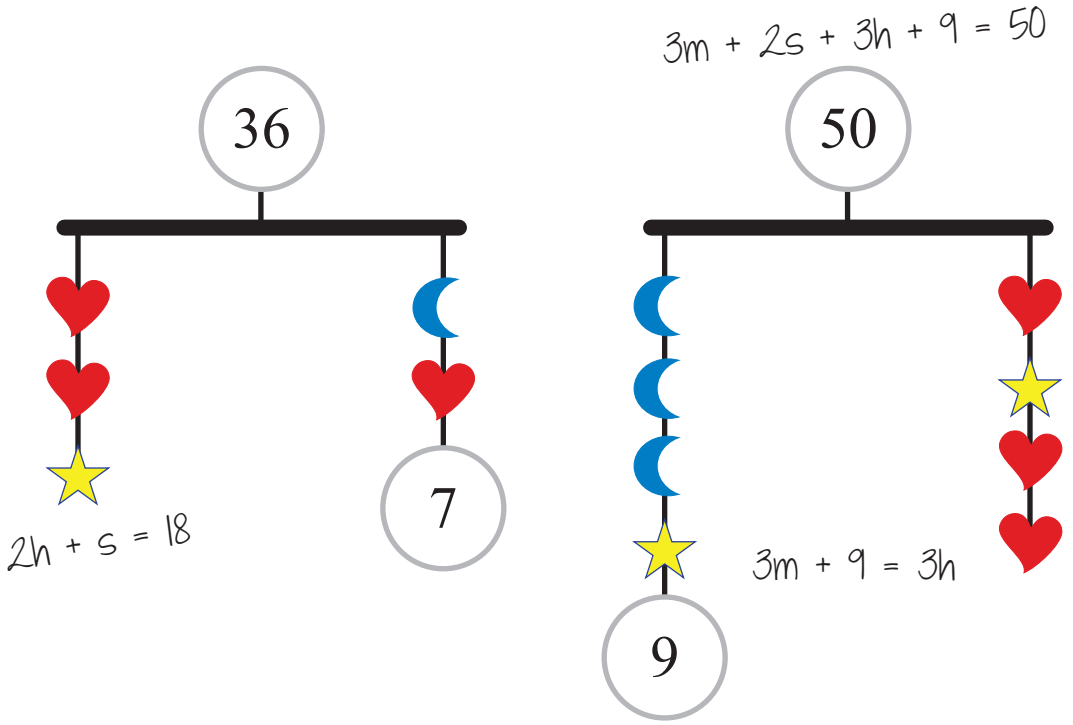
Ⓔ none of these

Use an area model to multiply these expressions.

⑩ $(x - 5)(y + 9) =$

⑪ $(n - 3)(n + 7) =$

Unit 5: Logic of Algebra



♥ = _____ ★ = _____ ☾ = _____

Transition to Algebra

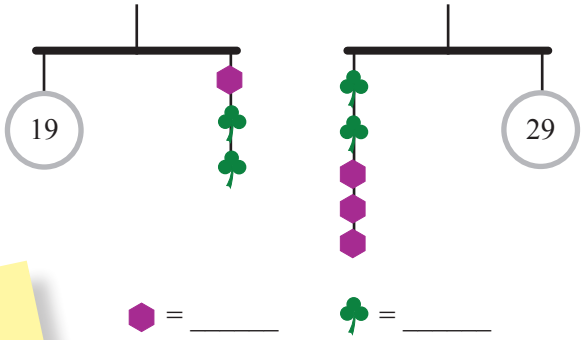
Unit 5: Logic of Algebra

Where's the Logic?

Algebra makes logical sense in the same way that mobiles make sense and number lines make sense and multiplying makes sense. Now is the time to connect them all together. In this Unit, you will revisit mobiles and you'll see a lot of equations - it may look like materials from any other algebra class. At every step, you can make sense of what you're doing. You'll find that complicated problems can be thought of in simpler ways, and the tools you've been using so far can really help you with algebra.

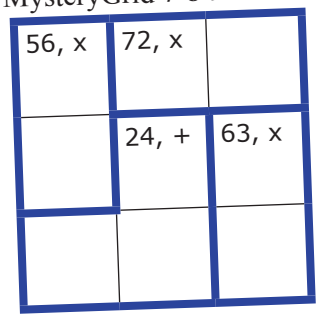
Lessons in this Unit:

1: Staying Balanced
2: Mobiles and Algebra
3: Keeping Track of Your Steps
4: Ordering Instructions
5: The Last Instruction
6: Solving Equations
7: Solving with Squares
8: Solving with Systems

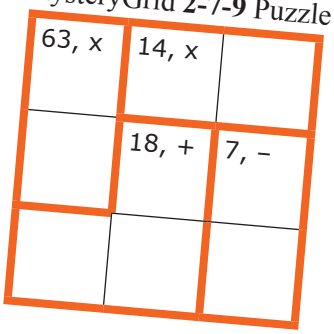


You will solve puzzles like these.

MysteryGrid **7-8-9** Puzzle



MysteryGrid **2-7-9** Puzzle



$\frac{16 - a}{7} + 52$

Think of a number.

Subtract it from 16.

Divide by 7.

Algebraic Habits of Mind: Using Structure

You will see that, sometimes, algebra problems can look exactly like mobile problems, and you can use the same strategies you used for mobiles to simplify and reorganize an equation.

Sometimes, an algebra equation looks complicated, but is really just a lot of layers of instructions, like in Think-of-a-Number puzzles. If you can figure out what the instructions are, you can start to unravel them until you arrive at the number in the bucket.

And sometimes, you see numbers that you recognize, but they're mixed in with expressions that look too complicated. You'll learn that sometimes it's good to leave those expressions as a complicated chunk to deal with later, and you'll learn how to use the numbers you recognize first.

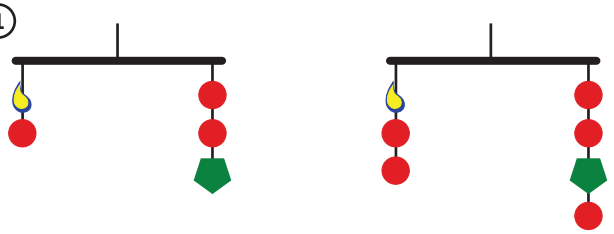
And through these structures, you'll start to see the logic in algebra.

5-1 Staying Balanced

In each problem, the first mobile is balanced. Use it to figure out whether that means the second mobile *for sure* has to balance. Circle your response and explain how you know your answer is right.

The only moves allowed on a mobile are moves that keep the mobile balanced.

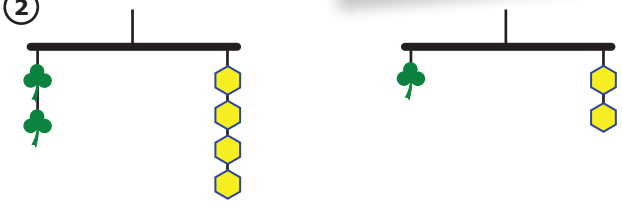
1



If this mobile balances... does this? YES or NO

Explain:

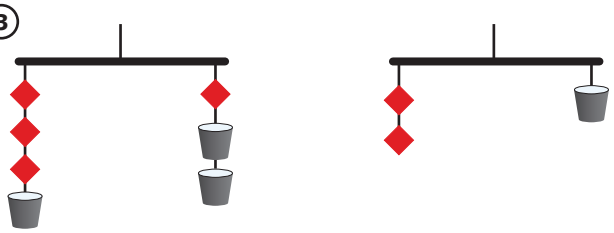
2



If this mobile balances... does this? YES or NO

Explain:

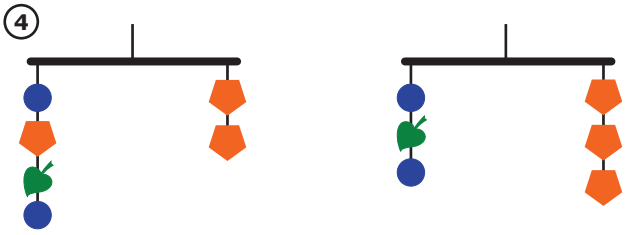
3



If this mobile balances... does this? YES or NO

Explain:

4

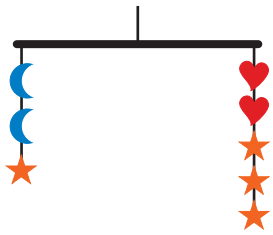


If this mobile balances... does this? YES or NO

Explain:

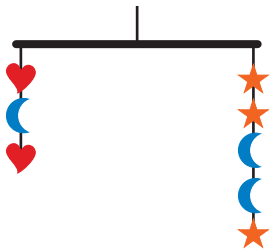
Translate each mobile into algebra. Let ☾ = m, ★ = s, and ♥ = h.

5



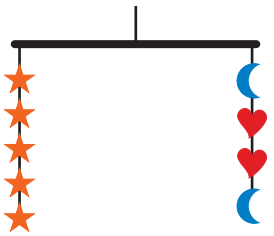
$2m + s =$ _____

6



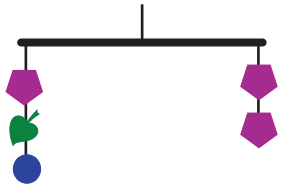
$h + m + h =$ _____

7



_____ = _____

8 Which of the following moves would keep the mobile balanced? Circle all that apply.



(A) Add a pentagon to both sides.

(B) Add 5 leaves to both sides.

(C) Move all the pentagons to the right side.

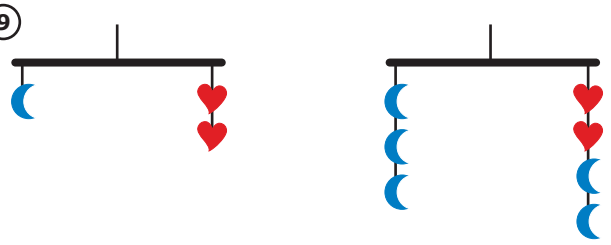
(D) Switch the leaf and circle.

(E) Add a circle to the right side.

(F) Remove one pentagon from both sides.

Translate each mobile into algebra or draw the mobile for the equation provided. Explain the change from the first to the second mobile and equation. Let ☾ = m, ★ = s, and ♥ = h.

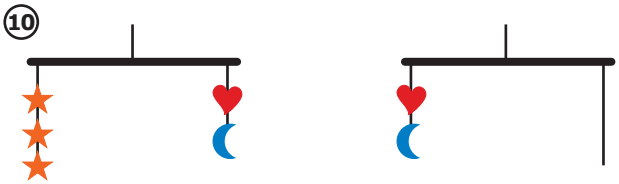
9



m = _____ _____ = _____

How did the mobile and the equation change?

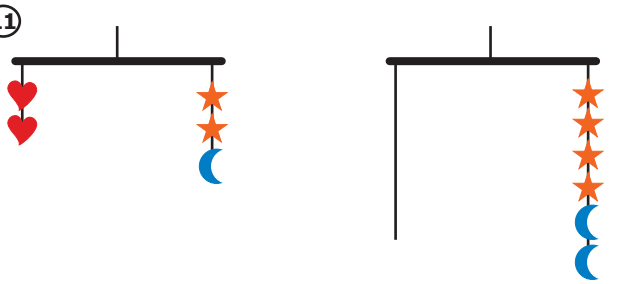
10



_____ = _____ _____ = $3s$

How did the mobile and the equation change?

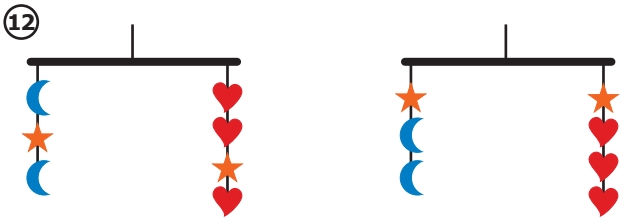
11



_____ = _____ $4h$ = _____

How did the mobile and the equation change?

12

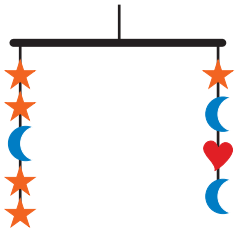


_____ = _____ _____ = _____

How did the mobile and the equation change?

13 This mobile balances. Translate it into an equation and circle all of the equations that must also be balanced.

m = ☾
s = ★
h = ♥



_____ = _____

(A) $2s + m + 2s = s + m + h + m$

(B) $4s + m = s + 2m + h$

(C) $m + 4s = 2m + s + h$

(D) $5s + m = 2m + h$

(E) $3s = h + m$

Hint: You should have circled four answers here. And in #8.

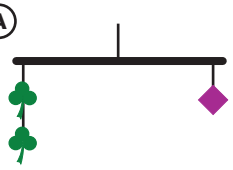
Discuss & Write What You Think

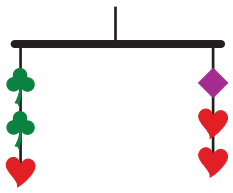
What moves are you allowed to make on a mobile or equation? What moves are you not allowed to make?

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.
In each problem, the first mobile is balanced. Use it to figure out whether that means the second mobile *for sure* has to balance. Circle your response and explain how you know your answer is right.

A

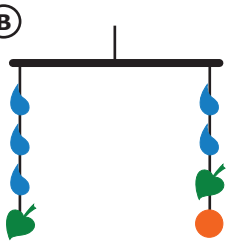




If this mobile balances... does this? YES or NO

Explain:

B

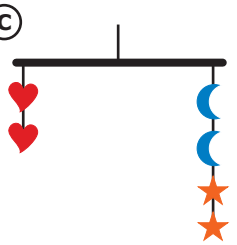


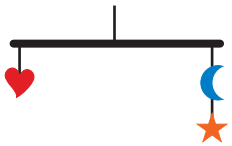
If this mobile balances... does this? YES or NO

Explain:

Translate each mobile into algebra or draw the mobile for the equation provided. Explain the change from the first to the second mobile and equation in words. Let ☾ = m, ★ = s, and ♥ = h.

C

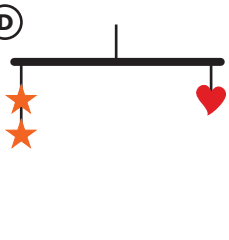


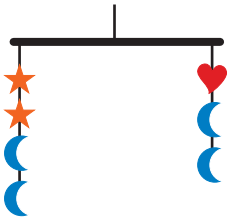


_____ = _____ _____ = _____

How did the mobile and the equation change?

D

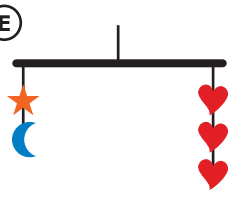


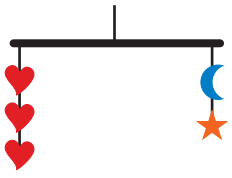


_____ = _____ _____ = _____

How did the mobile and the equation change?

E

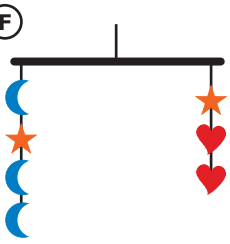


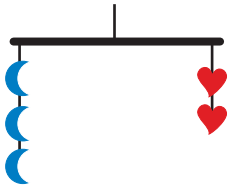


_____ = _____ _____ = _____

How did the mobile and the equation change?

F

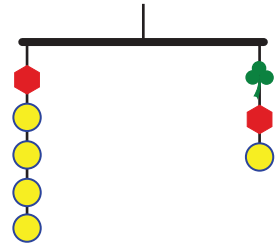




_____ = _____ _____ = _____

How did the mobile and the equation change?

G Which of the following moves would keep the mobile balanced? Circle all that apply.



(i) Add 3 circles to the right side.

(ii) Add 2 clovers (♣) to both sides.

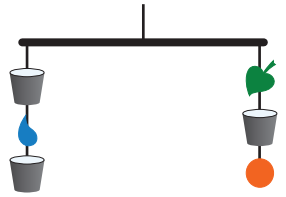
(iii) Move the shapes so all 5 circles are on the left side and both hexagons are on the right side.

(iv) Subtract a circle from both sides.

(v) Cross out both hexagons.

(vi) Subtract 3 circles from the left side.

H Which of the following moves would keep the mobile balanced? Circle all that apply.



(i) Add 4 water drops to both sides.

(ii) Remove a bucket from both sides.

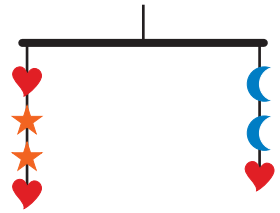
(iii) Move all the buckets to the left side and the water drop to the right side.

(iv) Add a bucket to both sides.

(v) Switch the leaf and the circle.

(vi) Switch the leaf and the water drop.

I Which of the following equations also must be balanced? Circle all that apply.



(i) $h + s + s + h = m + m + h$

(ii) $2s + h = 2m$

(iii) $6h + 6s = 6m + 3h$

(iv) $2m + h = h + 2s + h$

(v) $3s + 2h = 2m + h + s$

(vi) $3h = 2m + 2s$

Another hint:
G: 3 answers
H: 4 answers
I: 5 answers

J Fill in the blanks in this Think-of-a-Number Trick.

Instructions	Pictures	Result	Jacob	Mali	Kayla
Think of a number.					
Add 5.		b + 5		12	
Multiply by 2.			18		
Subtract 4.					12

K MysteryGrid **1-2-3-4** Puzzle

8, x	11, +		6, +
	12, x		4, x
3, ÷			

L MysteryGrid **1-3-4-5** Puzzle

4, +		4, ÷	1, −
20, x	12, +		
			2, −
	15, x		

M MysteryGrid **2-3-7-8** Puzzle

17, +	56, x	8, +	
			7
	18, x	4, ÷	
		1, −	

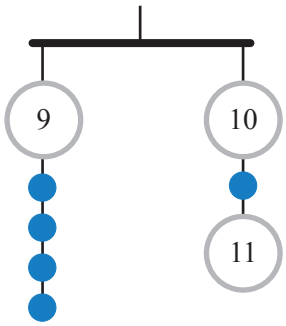
5-2 Mobiles and Algebra

Solve *both* the mobile and the equation, and check that you get the same answer.

The only steps that may be performed to an algebra equation are steps that keep the equation balanced.

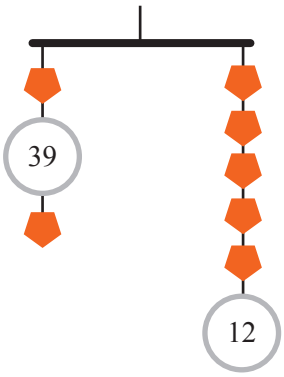
1 Find c... by using this mobile.

$9 + 4c = 10 + c + 11$



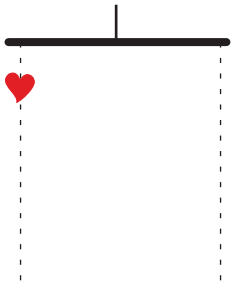
2 Find p... by using this mobile.

$p + 39 + p = 5p + 12$



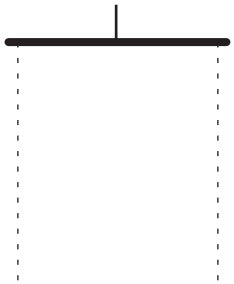
3 Find h... by drawing and using this mobile.

$h + 5 + 2h = 13 + h$

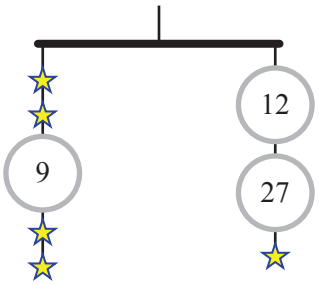


4 Find m... by drawing and using this mobile.

$15 + 2m = 2m + 9 + m$



5 Which of the following would work as a **first step** to solving this equation (use s = ☆)? Circle all that apply.



- **A** Subtract s from both sides.
- **B** Subtract 9 from both sides.
- **C** Add 12 and 27 so the right side becomes $39 + s$.
- **D** Move all the stars to the left and all the numbers to the right to get $5s = 48$.
- **E** Combine the stars on the left side to get $4s + 9 = 12 + 27 + s$.

Choose one of the first steps you circled, and use it as your first step to find s:

Find a friend who chose a **different** first step and compare your methods. Record their process here:

⑥ Which of the following would work as a **first step** to solving this equation? Circle all that apply.

$5b + 2 = 2b + 16 + b$

Find b, too.

$b =$ _____

Ⓐ On the right side, turn $2b + b$ into $3b$.

Ⓑ Subtract 2 from both sides.

Ⓒ Subtract $3b$ from both sides.

Ⓓ Combine all the b's and all the numbers to get $8b = 18$.

Ⓔ Subtract b from the right and add it to the left side to get $6b + 2 = 2b + 16$.

Ⓕ Subtract b from the left side and move it to the right to get the same number of b's on both sides: $4b + 2 = 4b + 16$.

Discuss & Write What You Think

Draw a mobile if it helps your explanation.

For problem 6, explain why...

choice d won't balance:

choice e won't balance:

choice f won't balance:

Algebraic Habits of Mind: Using Tools Strategically

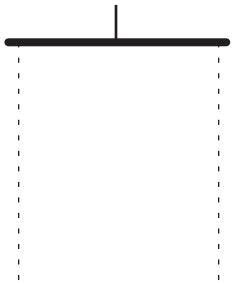
Mobiles are useful tools for visualizing balance. They help make sense of algebra, since solving algebra problems always requires the equation to stay balanced.

Mobiles are not *always* useful, however. For instance, how would you represent -5 on a mobile? Or $-x$? Or $\frac{1}{2}y$? Or $-6.27s$? You get the idea.

You can't draw a mobile for every algebra equation. But every equation balances. And anything you do to an equation has to keep the balance.

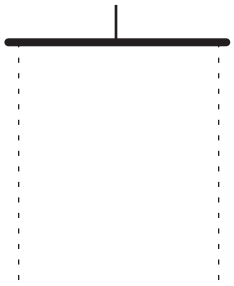
⑦ Find s... by drawing and using this mobile.

$10 + 4s = 8 + s + 11$



⑧ Find h... by drawing and using this mobile.

$h + 25 + h = 3h + 2h + 1$

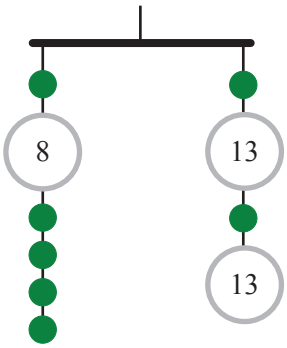


Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

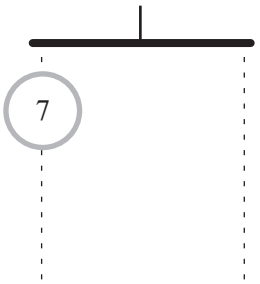
A Find c... by using this mobile.

$c + 8 + 4c = c + 13 + c + 13$



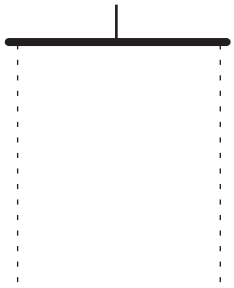
B Find h... by using this mobile.

$7 + h + 2h = 5 + 4h$



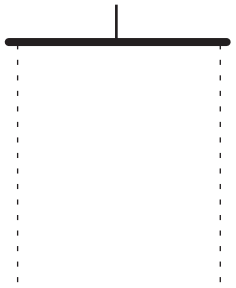
C Find s... by drawing and using this mobile.

$15 + 2s + 5 = s + 2 + 3s$

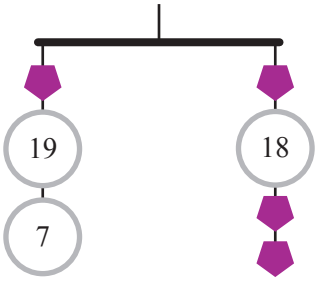


D Find m... by drawing and using this mobile.

$8 + m + 31 = 2m + 6 + 2m$



E Which of the following would work as a first step to solving this equation? Circle all that apply.



(i) Subtract 7 from both sides.

(ii) Add 19 + 7 + 18 = 44.

(iii) Subtract 18 from both sides.

(iv) Subtract a pentagon from both sides.

Find p, too.

$p =$ _____

F Which of the following would work as a first step to solving this equation? Circle all that apply.

$10 + 3d + 2 = 6 + 5d$

(i) Add 10 + 2 = 12 on the left.

(ii) Subtract $3d$ from both sides.

(iii) Subtract 6 from both sides.

(iv) Subtract 2 from both sides.

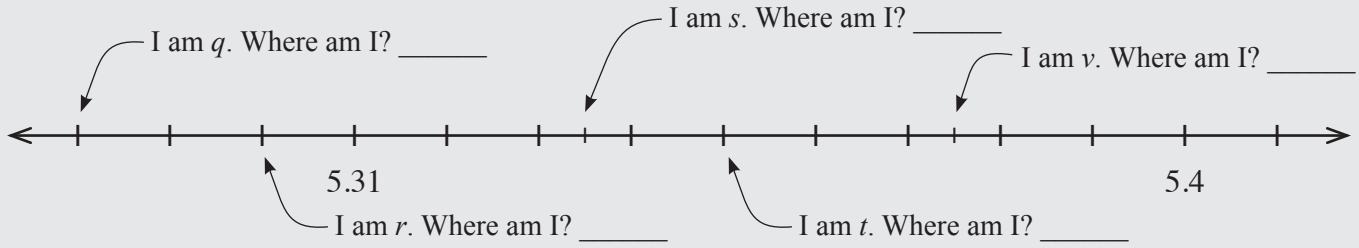
Find d, too.

$d =$ _____

G Fill in the blanks in this Think-of-a-Number Trick.

Instructions	Pictures	Result	Luis	Jess	Raj
Think of a number.					
Multiply by 3.					
Add 5.		$3b + 5$	26		
Multiply by 2.				-2	
Subtract 8.					20

H



Use an area model to multiply these expressions.

I

	100	___	___
___			
___			

$121 \cdot 34 =$ _____

J

	___	___
8		
-b		-3b

$(a + 3)(8 - b) =$

K

	2c	3
___		
___		

$(c + 9)(2c + 3) =$

L

	___	___	-7
___			
___			

$(d + 4)(d + y - 7) =$

M MysteryGrid **1-2-3-4** Puzzle

6, x		12, x	
	16, x		3, +
8, x	3		
		5, +	

N MysteryGrid **3-4-5-6** Puzzle

11, +		36, x	
4	15, x	10, +	
18, x			30, x
	20, x		

O MysteryGrid **5-6-7-8** Puzzle

1, –		3, –	
40, x	30, x	42, x	
		35, x	48, x
56, x			

5-3 Keeping Track of Your Steps

① Lena filled in a Think-of-a-Number table this way.

Instructions	Result
Think of a number.	b
Add 4.	$b + 4$
Multiply by 3.	$3b + 12$
Subtract 10.	$3b + 2$

Jay filled in the same Think-of-a-Number table this way.

Instructions	Result
Think of a number.	b
Add 4.	$b + 4$
Multiply by 3.	$3(b + 4)$
Subtract 10.	$3(b + 4) - 10$

Draw Lena and Jay's bucket pictures.

Instructions	Lena's Picture	Jay's Picture
Think of a number.		
Add 4.		
Multiply by 3.		
Subtract 10.		

Thinking out Loud

Lena: I guess we got the same answer. But I like my final result better. It's simpler!

Michael: I like Lena's final result better, too. $3b + 2$ is definitely simpler than $3(b + 4) - 10$. Why did you write it that way, Jay?

Jay: Actually, I like Lena's final result, too. But I was trying to make my final result *look* like a Think-of-a-Number trick.

Lena: What do you mean? Why doesn't mine look like a Think-of-a-Number trick?

Jay: Well, it does... just not the one in the problem. Look at my equation. Looking at $3(b + 4) - 10$, you can *see* the instructions, can't you? You can see that you start with b, then you first add 4, then multiply by 3, and then subtract 10.

Michael: Oh, I see! And Lena took the same steps to get to her final result, but you can't tell what those steps were. If you just saw her final result, you could have thought the instructions were...

Pausing to Think

Finish Michael's thought. If all you saw was $3b + 2$, what might you say were the Think-of-a-Number trick instructions?

Fill in the missing information on the following Think-of-a-Number tricks.

②

Instructions	Result
Think of a number.	c
	$2c$
	$2c - 3$
Multiply by 5.	$5(2c - 3)$
	$5(2c - 3) + 16$

③

Instructions	Result
Think of a number.	b
Add 4.	
Multiply by 3.	$3(\quad)$
Divide by 2.	$\frac{3(\quad)}{2}$
Subtract 9 from the result.	$\frac{\quad}{2} - 9$

④

Instructions	Result
Think	k
	$k - 2$
Divide by 3.	$\frac{k-2}{3}$
	$\frac{k-2}{3} + 25$
	$8(\frac{k-2}{3} + 25)$

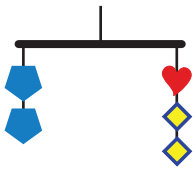
⑤

Instructions	Result
Think of a number.	w
Double that.	
Subtract that result from 9.	$9 -$
Multiply that by 3 (use parentheses).	
Add 8.	

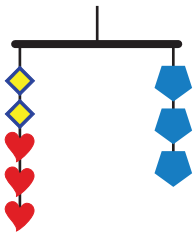
"Subtract 9" would be $2w - 9$. "Subtract from 9" means $9 - 2w$.

⑥

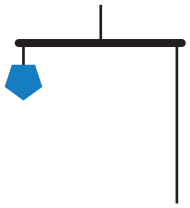
This mobile balances.



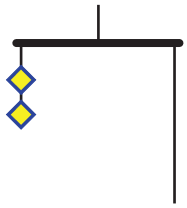
This mobile balances, too.



How many hearts will make this balance?



How many hearts will make *this* balance?



If ⬟ = 8, then what are the values of ♥ and ◇?

♥ =

◇ =

⑦

Instructions	Result
	m
	$\frac{m}{3}$
Add 11.	
	$10(\frac{m}{3} + 11)$
	$10(\frac{m}{3} + 11) - 1$

⑧

Instructions	Result
Think of a number.	y
Subtract 6.	
Multiply by 2.	
Add 5.	
Divide the result by 3.	

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.
Fill in the missing information on the following Think-of-a-Number tricks.

(A)

Instructions	Result
Think of a number.	n
	$9n$
	$9n + 1$
	$4(9n + 1)$
	$4(9n + 1) + 2$

(B)

Instructions	Result
Think of a number.	c
Multiply by 14.	
	$14c + 8$
Multiply the result by 5.	
Subtract 2 from the result.	

(C)

Instructions	Result
	d
	$d - 2$
Multiply by 13.	
	$13(d - 2) + 6$
	$\frac{13(d - 2) + 6}{15}$

(D)

Instructions	Result
Think of a number.	h
Add 5.	
Multiply the result by 2.	
Add 13.	
Divide by 9.	

(E)

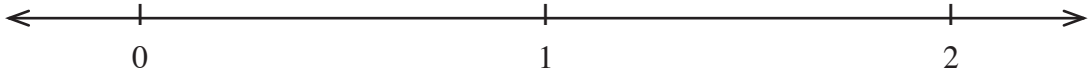
Instructions	Result
	p
	$p + 4$
Divide 20 by the result.	$\frac{20}{p + 4}$
	$\frac{20}{p + 4} - 1$
	$6(\frac{20}{p + 4} - 1)$

Divide 20 **by the result** means the result from the step above goes on the bottom.

(F)

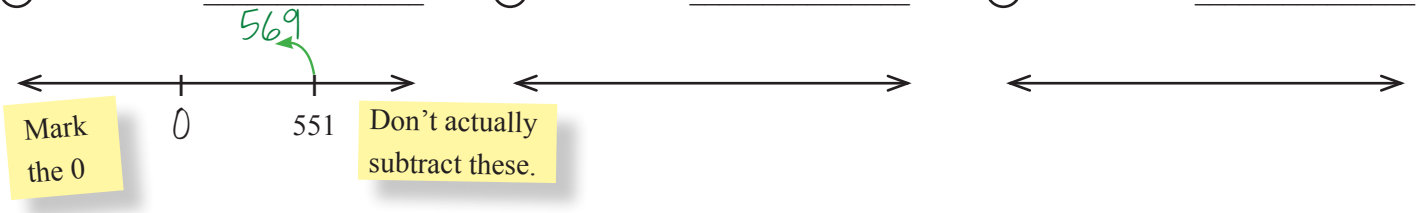
Instructions	Result
Think of a number.	k
Divide 40 by the result.	$\frac{40}{}$
Subtract 21.	
Multiply the result by 3.	
Add 81.	

(G) Place these numbers on the number line below: 1.5 1.05 0.5 0.05 $\frac{3}{4}$ $\frac{7}{8}$ $\frac{5}{4}$ $\frac{8}{8}$ $\frac{16}{8}$ $\frac{15}{8}$



Illustrate each problem on a number line. Then say *only* whether the answer will be **positive** or **negative**.

Ⓗ 551 – 569 is _____ Ⓘ 5.28 –5.8 is _____ Ⓙ -2.3 – -1.8 is _____

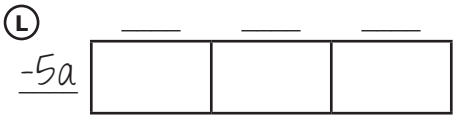


Ⓚ Who Am I?

- I am even.
- $u = h$
- I am greater than 200.
- My hundreds digit is a perfect square.
- $t = u + h$
- My tens digit is twice my hundreds digit.

h	t	u

Ⓛ



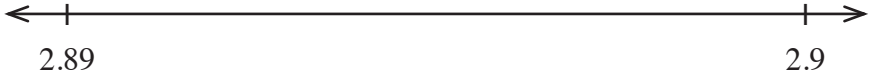
$-5a(w + 8 – a) =$

Ⓜ MysteryGrid **2-3-4-5** Puzzle

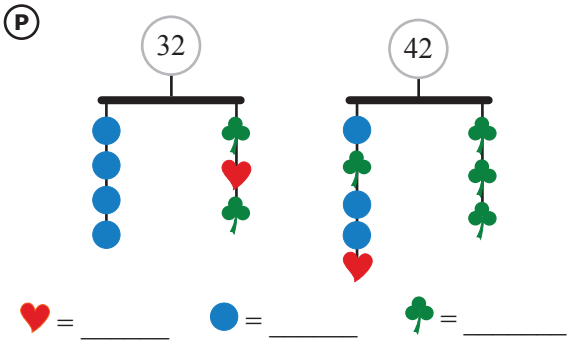
12, x		10, +	8, x
10, x			
	12, x		10, +
20, x			

Ⓝ I am exactly halfway between 8.2 and 8.7. Who am I? _____

Ⓞ I am exactly halfway between 2.89 and 2.9. Who am I? _____



Ⓟ

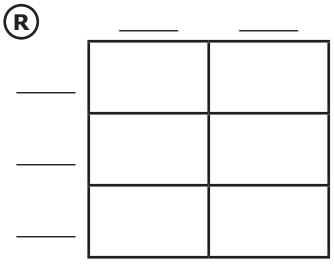


Ⓠ Who Am I?

- I'm not an even number.
- Two of my digits are even.
- My units digit is half my tens digit.
- All three of my digits are different.
- My hundreds digit is twice the sum of my units digit and my tens digit.

h	t	u

Ⓡ



$(n – 3p)(2n + 7p – 6) =$

5-4 Ordering Instructions

Arrange the steps below to create a trick that produces the final result. Fill in the table and the intermediate steps.

Add 14. | Divide by 7. | Multiply by 11. | Subtract 5.

(1)

Instructions	Result
Think of a number.	w
	$\frac{11w - 5}{7}$
	$\frac{11w - 5}{7} + 14$

(2)

Instructions	Result
Think of a number.	b
	$\frac{11(b + 14) - 5}{7}$

Divide by 8. | Subtract 22. | Add 13. | Multiply by 9.

(3)

Instructions	Result
Think of a number.	n
	$9(\frac{n + 13}{8} - 22)$

(4)

Instructions	Result
Think of a number.	c
	$9(\frac{c}{8} - 22) + 13$

Match each algebraic expression on the left with a set of instructions on the right.

(5) $\frac{4n + 18}{2}$

(6) $\frac{4n}{2} + 18$

(7) $\frac{2(n + 18)}{4}$

(8) $\frac{4(n + 18)}{2}$

(A) Think of a number.
Multiply by 4.
Divide by 2.
Add 18.

(B) Think of a number.
Multiply by 4.
Add 18.
Divide by 2.

(C) Think of a number.
Add 18.
Multiply by 4.
Divide by 2.

(D) None of the above

(10) $5(\frac{a + 10}{6})$

(11) $\frac{5(a + 10)}{6}$

(12) $\frac{5a}{6} + 10$

(13) $\frac{5a + 10}{6}$

(A) Think of a number.
Multiply by 5.
Add 10.
Divide by 6.

(B) Think of a number.
Add 10.
Divide by 6.
Multiply by 5.

(C) Think of a number.
Add 10.
Multiply by 5.
Divide by 6.

(D) None of the above

For each algebraic expression, write the instructions step-by-step, in the right order.

15 $\frac{10-n}{2} - 19$

Think of a number.

Subtract it from 10.

We're **not** subtracting 10. We're subtracting **from** 10.

16 $\frac{6(b-9)}{3}$

Th

17 $5(3-10q)$

Multiply it by ten.

We're **not** subtracting 3. We're subtracting **from** 3.

Match each algebraic expression on the left with a set of instructions on the right.

18 $\frac{-4(k+7)-14}{10}$

19 $-4(\frac{k+7}{10} - 14)$

20 $\frac{-4k+7}{10} - 14$

21 $-4(\frac{k}{10} + 7) - 14$

22 $\frac{7-4k}{10} - 14$

A Think of a number.
Add 7.
Divide by 10.
Subtract 14.
Multiply by -4.

B Think of a number.
Multiply by -4.
Add 7.
Divide by 10.
Subtract 14.

You can use B twice.

C Think of a number.
Divide by 10.
Add 7.
Multiply by -4.
Subtract 14.

D None of the above

23 $\frac{-8(m-3)+12}{5}$

24 $\frac{-8m-3}{5} + 12$

25 $12 - \frac{8(m-3)}{5}$

26 $\frac{-3-8m+12}{5}$

27 $\frac{-8(m-3)}{5} + 12$

A Think of a number.
Subtract 3.
Multiply by -8.
Add 12.
Divide by 5.

B Think of a number.
Subtract 3.
Multiply by -8.
Divide by 5.
Add 12.

Use B twice!

C Think of a number.
Multiply by -8.
Subtract 3.
Divide by 5.
Add 12.

D None of the above

28 MysteryGrid **4-5-6-7** Puzzle

28, x		30, x	
24, x	35, x	42, x	
		11, +	20, x
11, +			

29 MysteryGrid **1-2-3** Puzzle

6, x		18, x
2, x		

30 Who Am I?

- All of my digits are odd.
- $h + t + u = 23$
- $u - h = 4$
- $t \cdot h = 45$

h	t	u

31 Who Am I?

- None of my digits are odd.
- $h = 2u$
- $t = 2u$
- $u + t + h = 10$

h	t	u

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

For each algebraic expression, write the instructions step-by-step, in the right order.

A $5(c-4)+18$

Think of a number.

B $\frac{d-16}{3}+30$

C $-3(\frac{x}{8}+9)$

Match each algebraic expression on the left with a set of instructions on the right.

D $\frac{5(w-8)}{17}+6$

E $5(\frac{w}{17}-8)+6$

F $5(\frac{w-8}{17}+6)$

G $\frac{5(w-8)+6}{17}$

H $\frac{5w-8}{17}+6$

i Think of a number.
Subtract 8.
Multiply by 5.
Add 6.
Divide by 17.

ii Think of a number.
Divide by 17.
Subtract 8.
Multiply by 5.
Add 6.

iii Think of a number.
Subtract 8.
Divide by 17.
Add 6.
Multiply by 5.

iv None of the above

I $11(\frac{p+32}{8}-2)$

J $\frac{11(p+32)-2}{8}$

K $\frac{11p+32}{8}-2$

L $11(\frac{p}{8}+32)-2$

M $\frac{11(p+32)}{8}-2$

i Think of a number.
Multiply by 11.
Add 32.
Divide by 8.
Subtract 2.

ii Think of a number.
Add 32.
Multiply by 11.
Divide by 8.
Subtract 2.

iii Think of a number.
Add 32.
Divide by 8.
Subtract 2.
Multiply by 11.

iv None of the above

N Who Am I?

- My tens digit is odd.
- I am a multiple of 7.
- $t + 1 = u$
- u is not less than t.

t	u

O MysteryGrid **4-5-6** Puzzle

24, x	15, +	
		20, x
30, x		

P MysteryGrid **1-2-3** Puzzle

10, +		
18, x		

For each algebraic expression, write the instructions step-by-step, in the right order.

Q $\frac{5-y}{16} + 4$

Think of a number.

Divide by 16.

We're **not** subtracting 5. We're subtracting **from** 5.

R $\frac{19}{3a+2}$

Multiply

Divide 19 by the result.

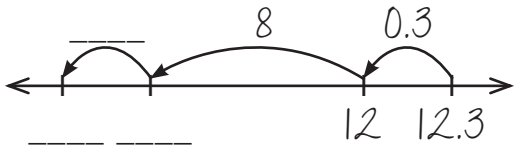
We're **not** dividing **by** 19.

S $-4(6-h)+15$

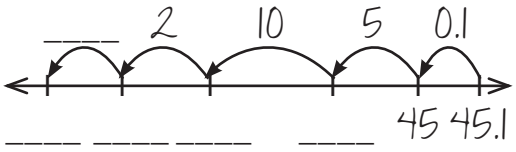
We're **not** subtracting 6.

Fill in the blanks on the number line illustrations and solve.

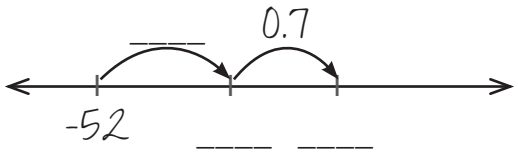
T $12.3 - 8.6 =$ _____



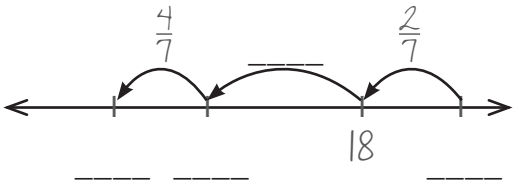
U $45.1 - 17.8 =$ _____



V $-52 - -4.7 =$ _____



W $18\frac{2}{7} - 2\frac{6}{7} =$ _____



X

____	____
____	
____	

$(7y-9)(6-y) =$

Y

____	____
____	x^2	8x
____	3x	24

(____ + ____) (____ + ____) $= x^2 + 11x + 24$

Z MysteryGrid **1-2-3-4-5** Puzzle

40, x		12, x	3, ÷	
	20, x		10, x	
6, x		8, x		
			1, –	9, +
	8, +			

5-5 The Last Instruction

These Think-of-a-Number puzzles have seen better days! Can you still fill in the LAST instruction?

①

Instructions	Result
Think of a number.	d
	$\frac{3(d+16)}{12} - 9$

②

Instructions	Result
Think of a number.	z
	$-5(\frac{7z-9}{36} - 14)$

Write the LAST instruction of each Think-of-a-Number trick. Use the words "the result" to refer to the rest of the expression.

③ $\frac{4x+3}{11}$ Divide the result by 11.

④ $9(m+2)$ _____

⑤ $8-3h$ from 8.

⑥ $\frac{14}{p+3}$ Divide 14 by the result.

⑦ $2(9r-4)-6$ _____

⑧ $\frac{k}{3}+4$ _____

⑨ $3+5(y-1)$ _____

⑩ $2(\frac{a-6}{5}+4)$ _____

⑪ $4-\frac{k-3}{2}$ _____

⑫ $\frac{5(p+2)-1}{3}$ _____

⑬ Make up an expression with the variable n and with at least 4 different numbers, where the LAST instruction would be "Add 23 to the result."

Check your answers with your classmates.

⑭ Who Am I?

- $t + u = 10$
- My tens digit is four more than my units digit.

t	u

⑮ Who Am I?

- My tens digit is 3 less than my units digit.
- $u + t = 7$

t	u

Match each algebraic expression on the left with a set of instructions on the right.

(16)	$8 - a$	(A)	8 less than a
(17)	$a - 8$	(B)	8 more than a
(18)	$a + 8$	(C)	a is less than 8
(19)	$a < 8$	(D)	a less than 8
(20)	$8 < a$	(E)	8 is less than a

(21)	$7 - b^2$	(A)	Subtract 7 from b and then square the result.
(22)	$(7 - b)^2$	(B)	Square b, then subtract 7.
(23)	$b^2 - 7$	(C)	Square b, then subtract the result from 7.
(24)	$(b - 7)^2$	(D)	Subtract b from 7, then square the result.

(25)	$2n - 10$	(A)	Subtract n from 10.
(26)	$n - 10$	(B)	Subtract 10 from n.
(27)	$10 - 2n$	(C)	Subtract 10 from twice n.
(28)	$10 - n$	(D)	Subtract 10 twice from n.
(29)	$n - 20$	(E)	Subtract twice n from 10.

(30)	$\frac{24}{d+5}$	(A)	Divide 24 by d, then add 5 to the result.
(31)	$\frac{24}{d} + 5$	(B)	Divide d by 24, then add 5 to the result.
(32)	$\frac{d}{24} + 5$	(C)	Add 5 to d, then divide by 24.
(33)	$\frac{d+5}{24}$	(D)	Add 5 to d, then divide 24 by the result.

Write an algebraic expression to match each sentence, using n for the unknown number.

(34) Four less than twice some number n.

_____

(35) Four minus twice some number n.

_____

(36) Subtract 3 from half some number n.

_____

(37) One less than the square of some number n.

_____

Algebraic Habits of Mind: Communicating Clearly

The details of language are important because talking about math will also help you think about and make sense of math.

(38) Sovann's age is twice that of his sister. If Sovann is 16, how old is his sister?

(39) Pengfei's age is twice that of his brother. If his brother is 16, then how old is Pengfei?

Discuss & Write What You Think

Wait a second. Did you get the same answer or different answers for problems 38 and 39? Which is it? Convince a skeptic.

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

Write the LAST instruction of each Think-of-a-Number trick. Use the words "the result" to refer to the rest of the expression.

$\frac{40}{7t-6}$ Divide 40 by the result.

What are you dividing **by**?

(A) $5h - 8$ _____

(B) $\frac{p-1}{10}$ _____

(C) $5 + 9(a - 6)$ _____

(D) $3(\frac{n}{2} + 4)$ _____

(E) $12 + \frac{6}{4-x}$ _____

(F) $8(2k - 1) - 3$ _____

(G) $7 - 4m$ _____

(H) $\frac{5 + 3(y-4)}{11}$ _____

(I) $16 - \frac{a+3}{15}$ _____

Match each algebraic expression on the left with a set of instructions on the right.

(J) $2(5 - b)$

(K) $2b - 5$

(L) $(5 - b)^2$

(M) $5 - 2b$

(N) $2(b - 5)$

(i) Subtract $2b$ from 5.

(ii) Subtract 5 from $2b$.

(iii) Subtract 5 from b, then double the result.

(iv) Subtract b from 5, then double the result.

(v) Subtract b from 5, then square the result.

(O) $\frac{y}{18} + 30$

(P) $\frac{18}{y+30}$

(Q) $\frac{y+30}{18}$

(R) $\frac{18}{y} + 30$

(i) Add 30 to y, then divide the result by 18.

(ii) Divide y by 18, then add 30 to the result.

(iii) Divide 18 by y, then add 30 to the result.

(iv) Add 30 to y, then divide 18 by the result.

Write an algebraic expression to match each sentence, using n for the unknown number.

(S) Think of a number n, divide it by 8, then add 2.

_____

(T) Think of a number n, multiply it by -4, then add 12 to the result.

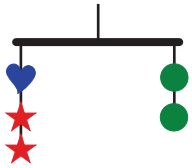
_____

(U) Think of a number n, subtract 3 from it, then multiply the result by -5.

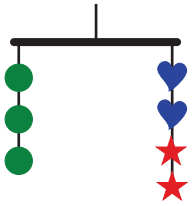
_____

Ⓥ Another Balancing Act

This mobile balances.



This mobile balances, too.

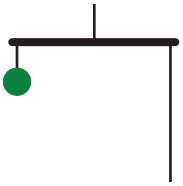


If ★ = 20, then what are the values of ● and ♥ ?

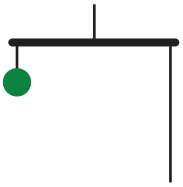
● =

♥ =

How many *hearts* will make this balance?



How many *stars* will make this balance?



Ⓦ Who Am I?

- My units digit is four more than my tens digit.
- $u > t$
- The product of my digits is 45.

t	u

Ⓧ Who Am I?

- My tens digit is 2 less than my units digit.
- $t + 2 = u$
- Neither of my digits is odd.
- $u + t = 10$

t	u

Write an algebraic expression to match each sentence, using n for the unknown number.

Ⓨ Think of a number n, square it, then subtract it from 10.

_____

Ⓩ Think of a number n, divide 9 by it, then square the result.

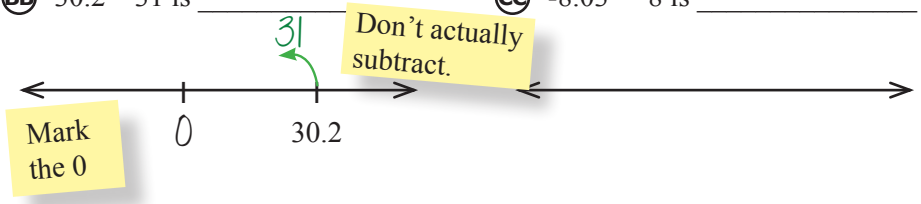
_____

ⒶⒶ Think of a number, n, subtract 2 from it, then divide by 9.

_____

Illustrate each problem on a number line. Then say *only* whether the answer will be **positive** or **negative**.

ⒷⒷ 30.2 – 31 is _____



ⒸⒸ -8.03 – -8 is _____

ⒹⒹ 2.18 – 2.3 is _____

ⒺⒺ MysteryGrid **1-3-5-7** Puzzle

3, x	15, +		
	49, x		5, ÷
15, +			
	5, x		3

ⒻⒻ MysteryGrid **2-4-6-8** Puzzle

12, x		12, +	
24, x	2, ÷	12, x	
		2, –	6, +
16, x			

ⒼⒼ MysteryGrid **1-2-3-4** Puzzle

3, ÷		8, x	
6, x	16, x	3	
			6, x
7, +			

5-6 Solving Equations

Thinking out Loud

Michael: Here's a Think-of-a-Number puzzle: Think of a number, multiply by 2, and subtract your result from 11. I got 5. Can you figure out my original number?

Lena: Well, I like to think of it as an equation.

(Lena writes:) $11 - 2x = 5$

Then we solve for x.

Pausing to Think Use Lena's equation to try to find Michael's original number.

Jay: When I look at that equation, I don't just focus on the x. Instead, I look at the bigger picture. That equation is really just saying "11 minus something equals 5." And we know *that* answer!

Michael: Well, 11 minus 6 is 5. How does that help?

Jay: Watch. Let me write down what we know under the equation.

(Jay writes:) $11 - 2x = 5$

$11 - \ 6 \ = 5$

See? They're almost the same thing.

The only thing that's different is that...

Lena: $2x$ is 6. Oh! I get it! $2x$ is 6! So I can write:

$2x = 6$

Michael: And if $2x = 6$, then x must be 3.

And that was my original number!

Algebraic Habits of Mind: Chunking

Looking at the bigger picture can help you make sense of a complicated equation. Even if the equation has lots of steps and numbers and symbols, if you can see that

$11 -$ **something** $= 5$,

at least you know that "something" must equal 6.

Solve each equation.

① $20 - \blacksquare = 2$

What is $\blacksquare$?

② $20 - 3p = 2$

That's $20 - 3p = 2$

$3p =$ ____

$p =$ ____

③ $20 - (a + 10) = 2$

④ $\frac{30}{\blacksquare} = 3$

What is $\blacksquare$?

⑤ $\frac{30}{b+1} = 3$

$b + 1 =$ ____

$b =$ ____

⑥ $\frac{30}{5y} = 3$

Solve these equations. Show your process.

⑦ $16 - (p + 1) = 11$

That's just $16 - $ [redacted] $= 11$. So what's p?

$p + 1 = \_\_\_\_$

$p = \_\_\_\_$

⑧ $\frac{45}{y + 3} = 9$

$y + 3 = \_\_\_\_$

$y = \_\_\_\_$

⑨ Think of a number, subtract 8, multiply by 6, and add 5. Imani followed the instructions and got 29 as her final result. What number did she think of?

⑩ $2 + 3(y - 1) = 23$

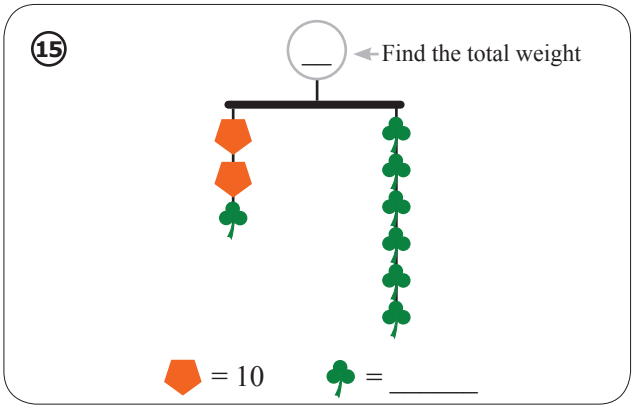
⑪ $\frac{24}{2n - 10} = 3$

⑫ Think of a number, multiply by 3, add 10, then double the result. Jacob followed the instructions and got 62 as his final result. What number did he think of?

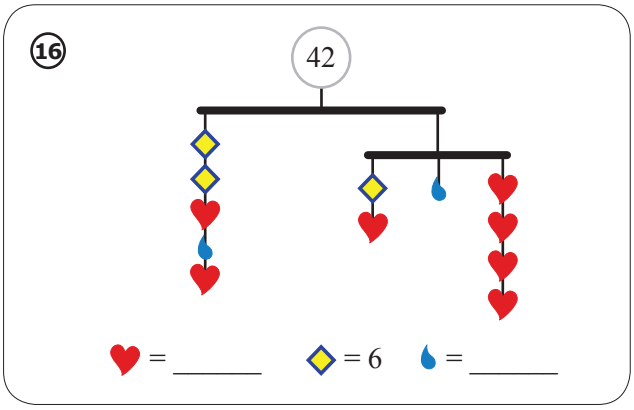
⑬ $15 - \frac{21}{a + 3} = 12$

⑭ $\frac{3h + 5}{5} = 7$

⑮



⑯



Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

Solve these equations. Show your process.

A $10 - 3x = 4$

$3x =$ ____

$x =$ ____

B $5 + (m - 2) = 20$

C $\frac{25}{a+3} = 5$

$a + 3 =$ ____

$a =$ ____

D $\frac{p+4}{6} = 10$

E $20 - (y + 6) = 10$

F $\frac{100}{2b-6} = 25$

$2b - 6 =$ ____

$2b =$ ____

$b =$ ____

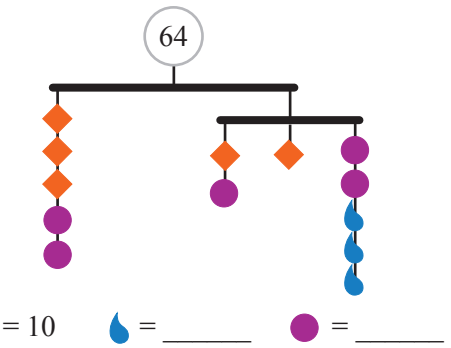
G $7(10 - k) + 4 = 11$

H $8 - \frac{10}{h} = 3$

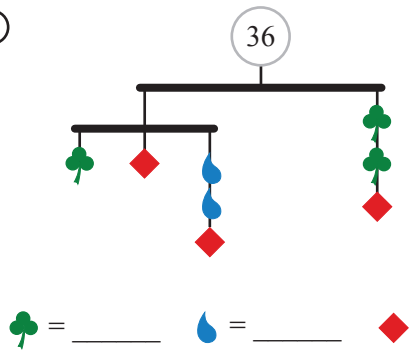
I $1 + 2(3c + 5) = 47$

J $9 - \frac{12}{10 - 8y} = 3$

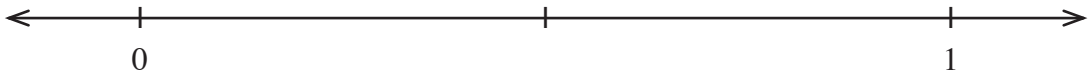
K



L



M Place these numbers on the number line below: 0.5 0.25 0.75 $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$



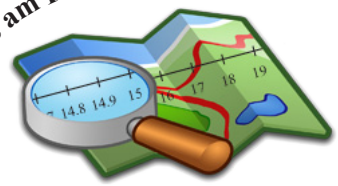
Write an algebraic expression to match each sentence, using *n* for the unknown number.

N Subtract a number *n* from 15. _____

O 15 minus a number *n*. _____

P 15 less than a number *n*. _____

Where am I?



Q I am exactly halfway between 27 and 43. Who am I? _____

R I am exactly halfway between -0.02 and 0.08. Who am I? _____

S MysteryGrid **1-2-3-4** Puzzle

2, –	1	12, x	
	24, x		1, –
12, x			
		6, x	

T MysteryGrid **3-6-9** Puzzle

18, x		15, +
54, x	15, +	

U

___	a^2	9a
___	7a	63

(____ + ____) (____ + ____) = $a^2 + 16a + 63$

5-7 Solving with Squares

Thinking out Loud

Michael: If $n^2 = 36$, then there are two possible numbers that n can be: either 6 or -6. But what happens when something more complicated is squared?

Lena: I saw a problem like that the other day. It was... oh yes.

(Lena writes:) $(p - 5)^2 = 36$

Michael: Something is still squared, but this time it's $p - 5$. But that would mean that $p - 5$ could be -6 or 6. How would we write that?

Jay: Why not with two equations? Write what you just said:

(Jay writes:) $p - 5 = 6$ or $p - 5 = -6$

Michael: And so we get two possible answers for p. It must either be 11 or -1... So which one is it?

Lena: Both! They both work in the equation.

Pausing to Think Show that both 11 and -1 work in the equation $(p - 5)^2 = 36$.

Jay: There are two answers! Just like there are two answers for $n^2 = 36$. That makes sense. The only two numbers that work in this equation are 11 and -1.

Algebraic Habits of Mind: Chunking

We're chunking again! But this time, something^2 = 36, so you know that "something" must equal -6 or 6!

Solve the following. Show your steps.

1 $(c + 3)^2 = 64$

That's something^2 = 64. So what's c?

$c + 3 =$ ____ OR $c + 3 =$ ____

$c =$ ____ OR $c =$ ____

2 $(y - 1)^2 = 49$

3 $(10 - n)^2 = 81$

4 $(4x + 2)^2 = 100$

5 $2(h + 3)^2 = 50$

$(h + 3)^2 =$ ____

$h + 3 =$ ____ OR $h + 3 =$ ____

$h =$ ____ OR $h =$ ____

6 Think of a number, subtract 5, then square the result. If you end up with 16, what are the only two numbers you could have started with?

Secret Code Numbers: All the shapes represent one-digit numbers, **0 through 9**.

⑦ What are the only two numbers that ● could be if ● • ● = ● ?

● = _____

Consider only the digits 0–9.

⑧ What is the only number that ♥ could be if ♥ + ♥ = ♥ ?

♥ = ____

⑨ What are the numbers that ⬟ could be if ⬟ • ⬟ = ♣ ? Then what could ♣ be?

⬟ = _____

♣ = _____

⑩ Let's look at two equations at once. What could ⬟ and ♣ be?

⬟ • ⬟ = ♣

⬟ + ⬟ = ♣

⬟ = _____

♣ = _____

⑪ What could ⬟, ♣, and ☾ be if all the shapes are different numbers?

⬟ • ⬟ = ♣

⬟ + ⬟ = ☾

⬟ = _____

♣ = _____

☾ = _____

The numbers have to work for **all** the equations in a problem.

⑫ What could ◆ be? Don't try to find 🪣. (Why not?)

◆ • ◆ = ◆

◆ + 🪣 = 🪣

◆ = _____

⑬ What could ☆, 💧, and ⬢ be if all the shapes are different numbers?

💧 • ⬢ = ☆

⬢ + ⬢ = ☆

💧 + 💧 + 💧 = ☆

☆ = _____

💧 = _____

⬢ = _____

⑭ What could ♥, 🍃, and ◆ be if all the shapes are different numbers?

♥ • 🍃 = 🍃

🍃 • 🍃 = ◆

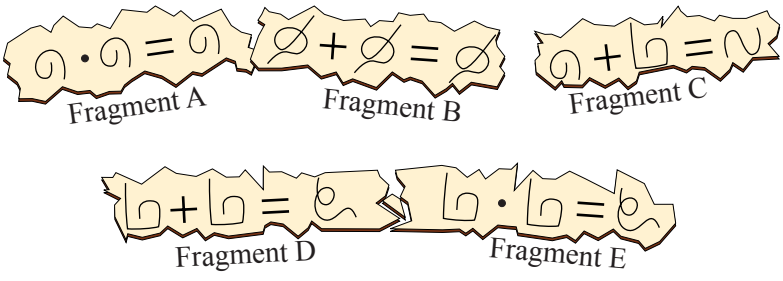
🍃 + 🍃 = ◆

♥ = _____

🍃 = _____

◆ = _____

⑮ You found several fragments of pottery—maybe a secret code, or from a space ship or an ancient ruin, or just from some country whose language you don't know. From other fragments, you already figured out that the "•" and "+" and "=" mean exactly what they mean in your language, and it turns out that the other symbols are digits, all **different** numbers from 0 through 9. But *what* digits?!



ก = _____ ∅ = _____ ใ = _____ ∩ = _____ ๖ = _____

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.
Solve the following. Show your steps.

A $(a + 10)^2 = 144$

$a + 10 =$ ____ OR $a + 10 =$ ____

$a =$ ____ OR $a =$ ____

B $(w - 8)^2 = 81$

C $(13 - b)^2 = 25$

D $(2n - 1)^2 = 81$

E $(h - 14)^2 + 3 = 28$

$(h - 14)^2 =$ ____

F $20 - (m + 1)^2 = 4$

$(m + 1)^2 =$ ____

$m + 1 =$ ____ OR $m + 1 =$ ____

$m =$ ____ OR $m =$ ____

G $5(12 - x)^2 = 45$

H $\frac{(b + 9)^2}{2} = 8$

I You have found these calculations carved on a piece of rock in your backyard. You know that "•", "+", and "=" are exactly like our algebra and that the other symbols are all different digits (integers 0-9). What are the digits?

$$\mathrm{T} \bullet \omega = \omega$$
$$\mathrm{T} + \mathrm{T} = \omega$$
$$\mathrm{T} + \omega = \mathrm{M}$$
$$\mathrm{M} + \mathrm{M} = \mathrm{O}$$
$$\mathrm{M} \bullet \mathrm{M} = \mathbb{S}$$

T = ____

ω = ____

M = ____

O = ____

$\mathbb{S}$ = ____

Solve the following. Show your steps!

(J) $16 - (c + 8)^2 = 15$

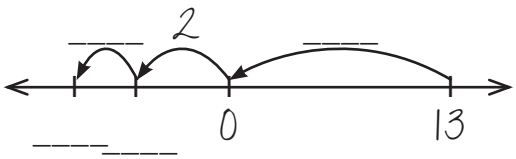
(K) $4(10 - k)^2 - 3 = 13$

(L) Fill in the blanks in this Think-of-a-Number Trick.

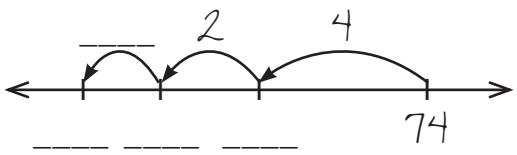
Instructions	Result	Jacob	Mali	Kayla
Think of a number.	n			
Add 6.				
Multiply by 2.		24		
Subtract 4.	2(n + 6) - 4		16	10

Fill in the blanks.

(M) $13 - 15.6 =$ _____



(N) $74 - 6.3 =$ _____



(O)

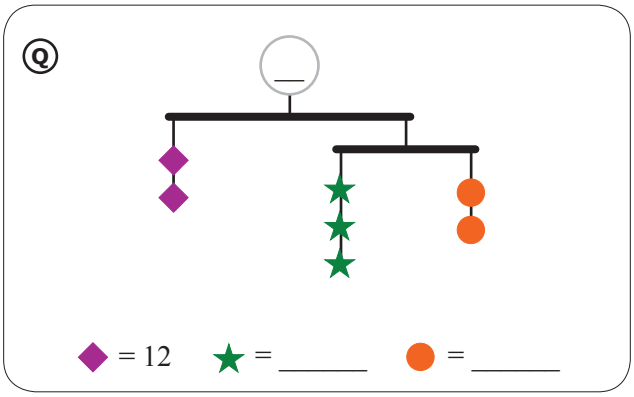
$(m + 5)(4m - 7 - 3m) =$

(P)

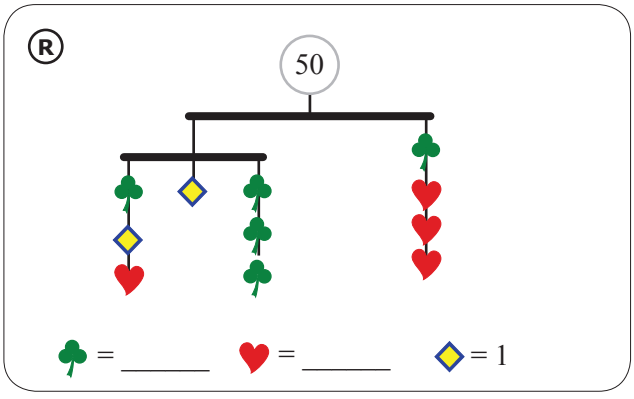
	x^2	$9x$
	$4x$	36

(____ + ____) (____ + ____) = $x^2 + 13x + 36$

(Q)

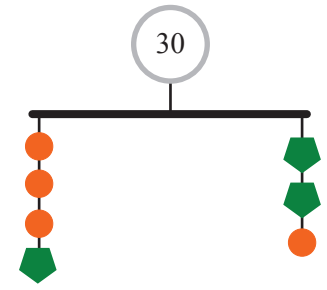


(R)



5-8 Solving with Systems

1



● = _____ ⬟ = _____

Solve the mobile. You can use the equations you wrote if you would like, but you don't have to.

Let c stand for ● and let p stand for ⬟.

There are **many** equations that you can write using the mobile.

Write two different equations:

_____

_____

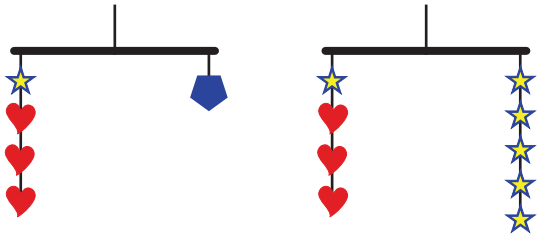
Share your equations and your strategies for solving.

Write two more true equations that other classmates wrote:

_____

_____

2 Write two different equations using these two mobiles. Let p stand for ⬟, let s stand for ☆, and let h stand for ♥.



⬟ = 30 ☆ = _____ ♥ = _____

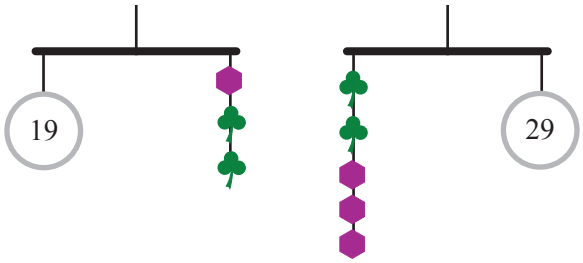
Solve the mobiles, too.

Equations:

_____

_____

3 Write two different equations using these two mobiles. Let h stand for ⬢ and let c stand for ♣.



⬢ = _____ ♣ = _____

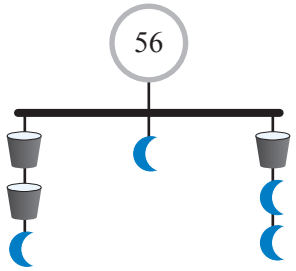
Solve the mobiles, too.

Equations:

_____

_____

4 Write two different equations using this mobile. Let b stand for 🪣 and let m stand for ☾.



🪣 = _____ ☾ = _____

Solve the mobiles, too.

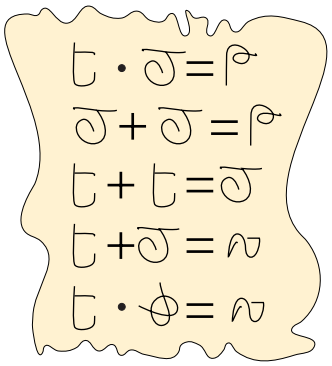
Equations:

_____

_____

⑤ More Mystery Numbers!

You found these calculations painted on an old piece of leather.
You know that "•" ("times"), "+" and "=" are exactly like our algebra.
You've also figured out that the other symbols are all different digits.



You have to look at two or more of these equations at a time to figure out anything.

t = _____ p = _____ ϕ = _____ ϑ = _____ n = _____

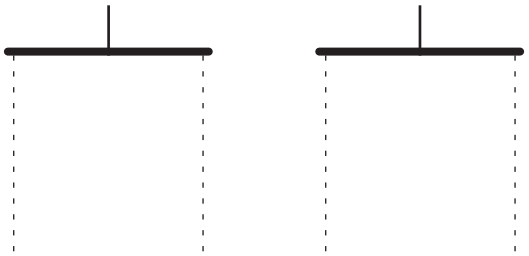
⑥ Use the two equations to draw one or two mobiles relating *h* (♥) and *s* (☆).

You may use one or both mobiles.

Equations:

$2h + s = 10$

$5h = 10$



♥ = _____ ☆ = _____

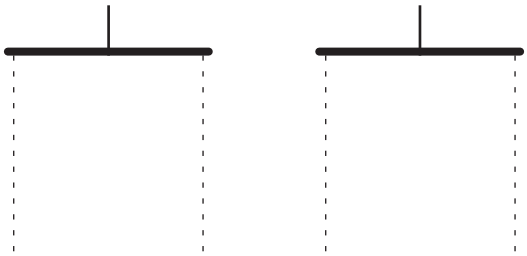
Solve the mobiles, too.

⑦ Use the two equations to draw one or two mobiles relating *b* (🪣) and *c* (●).

Equations:

$b + c = 11$

$2b + 3c = 24$



🪣 = _____ ● = _____

Solve the mobiles, too.

⑧ Who Am I?

- I'm odd.
- $h + t + u = 3 \cdot 3 \cdot 3$
- My units digit is not greater than my tens digit.
- My hundreds digit is not less than my tens digit.
- $t > 8$

h	t	u

⑨ Who Am I?

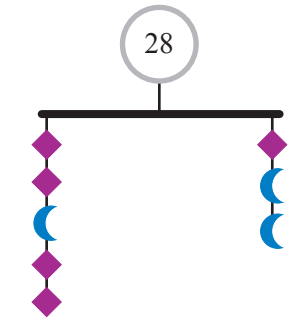
- I'm odd.
- $h + t + u = 6$
- My units digit > my tens digit.
- $h > t$
- $t + u = h$
- The product of my three digits is 0.

h	t	u

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

A



◆ = _____ ☾ = _____

Solve the mobile, too.

Let d = ◆ and let m = ☾ .

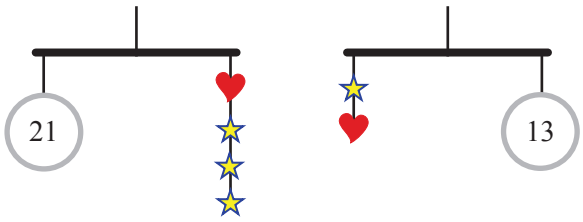
There are **many** equations that you can write using the mobile.

Write two different equations:

_____

_____

B Write two different equations using these two mobiles. Let h = ♥ and let s = ☆.



♥ = _____ ☆ = _____

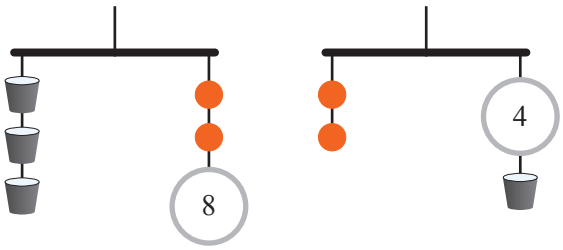
Solve the mobiles, too.

Equations:

_____

_____

C Write two different equations using this mobile. Let b stand for 🪣 and let c stand for ●.



🪣 = _____ ● = _____

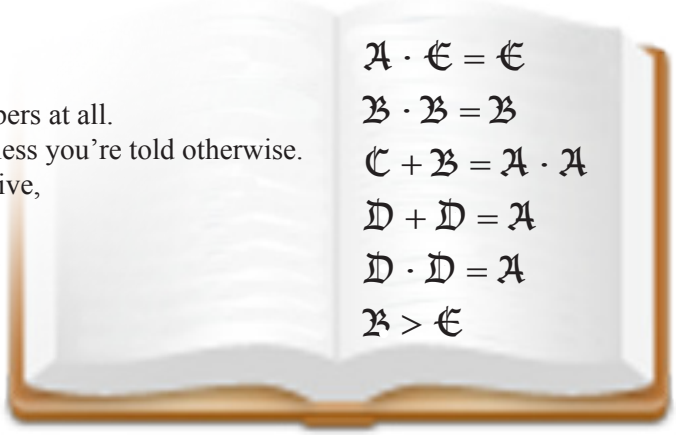
Solve the mobiles, too.

Equations:

_____

_____

D You found this puzzle in a very 𝔬𝔩𝔡𝔢 algebra book.
Because it's algebra, these letters can stand for *any* numbers at all.
Different letters can even stand for the *same* number, unless you're told otherwise.
In this problem, no mystery number happens to be negative,
but at least one of them is greater than 10.
What can you figure out?



𝔄 = _____ 𝔅 = _____ ℭ = _____ 𝔇 = _____ 𝔈 = _____

Match each algebraic expression on the left with a set of instructions on the right.

E $\frac{5(m+4)-8}{3}$

F $5(\frac{m+4}{3}-8)$

G $5(\frac{m}{3}+4)-8$

H $\frac{5(m+4)}{3}-8$

I $\frac{5m+4}{3}-8$

i Think of a number.
Add 4.
Divide by 3.
Subtract 8.
Multiply by 5.

ii Think of a number.
Add 4.
Multiply by 5.
Subtract 8.
Divide by 3.

iii Think of a number.
Multiply by 5.
Add 4.
Divide by 3.
Subtract 8.

iv None of the above

J $h-9$

K $h<9$

L $2h-9$

M $9-h$

N $9-2h$

i h less than 9.

ii Subtract 9 from h.

iii 9 less than twice h.

iv 9 minus twice h.

v h **is** less than 9.

O Who Am I?

- All of my digits are even.
- My units digit is the product of my tens digit and my hundreds digit.
- My hundreds digit is less than my tens digit.

h	t	u

P

____	8
____	x^2
9	72

(____ + ____) (____ + ____) = $x^2 + 17x + 72$

Q

____	____
____	
____	

$(7b + 2c)(5c - 8b) =$

R

____	____
____	
____	
____	

$(x + y + z)(3y - x) =$

S MysteryGrid **1-3-5-7** Puzzle

21, x	7, x		15, +
	25, x		
2, –			
12, +		3, ÷	

T MysteryGrid **2-4-6-8** Puzzle

2, –		48, x	
16, x	3, ÷		32, x
	24, x		
2, –		2, ÷	

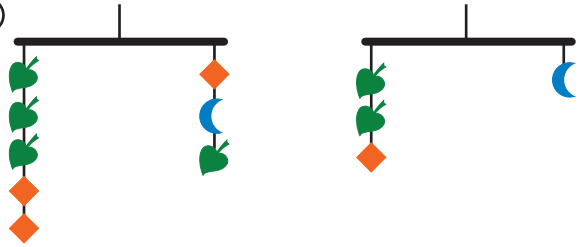
U MysteryGrid **6-7-8-9** Puzzle

1, –		1, –	
15, +	54, x		13, +
	8	63, x	
15, +			8

Unit Additional Practice Problems

Use these questions to help you prepare for the Unit Assessment.

1

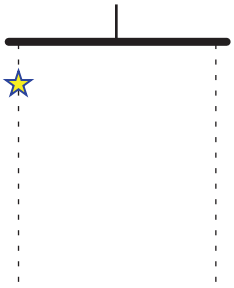


If this mobile balances... does this? YES or NO

Explain:

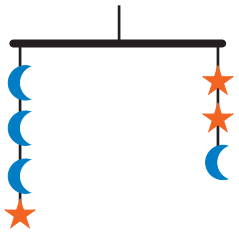
2 Find s (☆)... by drawing and using this mobile.

$2s + 10 + 6 = 2 + 4s$



3 This mobile balances. Translate it into an equation and circle all of the equations that must also be balanced.

$m =$ ☾
$s =$ ★



_____ = _____

(A) $3m + s = m + 2s$

(B) $4m = 3s$

(C) $2m + s = 2s$

(D) $2m = s$

(E) $3m = m + s$

Match each algebraic expression on the left with a set of instructions on the right.

4 $9(\frac{b}{2} - 3) + 7$

5 $\frac{9(b-3)+7}{2}$

6 $\frac{9b-3}{2} + 7$

7 $9(\frac{b-3}{2} + 7)$

8 $\frac{9(b-3)}{2} + 7$

(A) Think of a number.
Subtract 3.
Divide by 2.
Add 7.
Multiply by 9.

(B) Think of a number.
Subtract 3.
Multiply by 9.
Add 7.
Divide by 2.

(C) Think of a number.
Divide by 2.
Subtract 3
Multiply by 9.
Add 7.

(D) None of the above

9 $y - 5$

10 $5 - 2y$

11 $2y - 5$

12 $5 - y$

(A) Subtract 5 from y.

(B) 5 minus y.

(C) Subtract twice y from 5.

(D) 5 less than twice y.

13 What could ●, 🍃, and ◇ be if all the shapes are different numbers?

🍃 • 🍃 = ●

🍃 + 🍃 = ◇

● = _____

🍃 = _____

◇ = _____

14

Instructions	Result
Think of a number.	n
Divide by 2.	
Add 8.	
Multiply by 10.	
Subtract the result from 15.	

Solve these equations. Show your process!

15. $20 - (h + 5) = 8$

16. $\frac{48}{3x} = 8$

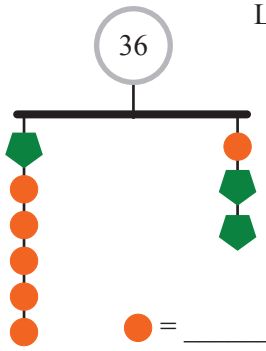
17. $5(p - 1) + 15 = 30$

18. $\frac{18}{m - 10} = 6$

19. $(a + 5)^2 = 49$

20. $(n - 8)^2 = 100$

21.



Let $c =$ ● and let $p =$ ⬟.

Write two equations:

_____

_____

● = _____ ⬟ = _____

Solve the mobile, too.

22. $\frac{4a + 5}{3} - 1$

Write the instructions in order.

Think

_____

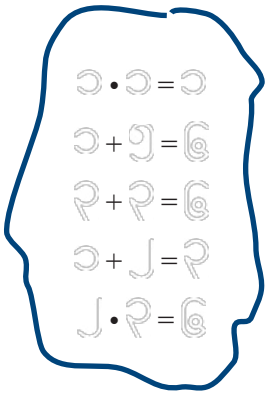
_____

_____

_____

_____

24. Your neighbor also found calculations carved on pieces of rock in his backyard. You want to help him find out what they mean. You know that "•", "+", and "=" are exactly like our algebra. You've also figured out that the other symbols are all different digits (integers 0-9). Can you help your neighbor find the digits?



○ = _____

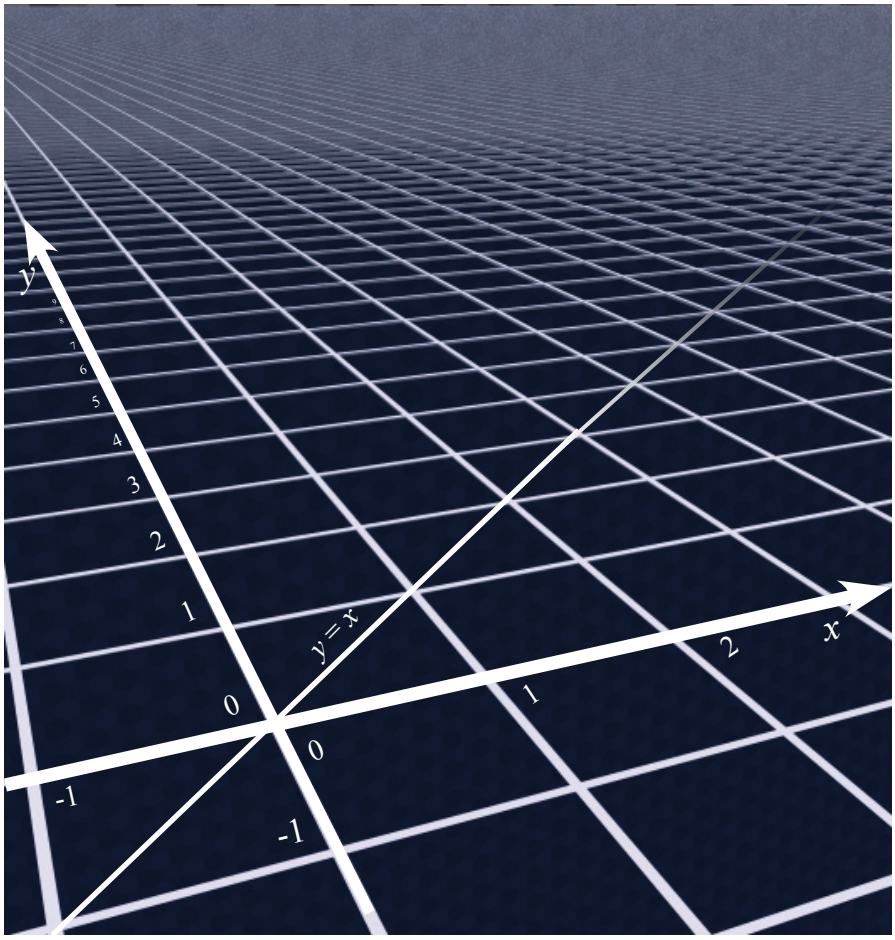
△ = _____

□ = _____

? = _____

◎ = _____

Unit 6: Geography of the Coordinate Plane



Transition to Algebra

Unit 6: Geography of the Coordinate Plane

Geography of the Coordinate Plane

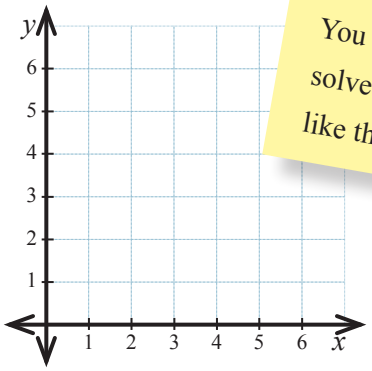
X marks the spot! If the spot we are looking for is on a line – a highway, a railroad track, or a number line – we can say exactly where it is with just one number. Because one number is enough to specify any point on that line, we call a line "one-dimensional."

If "X marks the spot" on a surface – the surface of the earth, a map, a piece of paper, or an entire coordinate plane – we need two numbers to say where it is in the two-dimensional space. One way of doing that uses two number lines, crossing each other at right angles. The two measurements on those two number lines don't need to be about "location on a map," either. The graph can be used to coordinate any two pieces of numeric information you have and can help you see patterns and relationships in that information.

Lessons in this Unit:

1: Plotting Data
2: Coordinating Data
3: Geometric Transformations
4: Transformations with Algebra
5: Intuitive Graphing
6. Where Could I Be? Solutions and Point Testing
7. Graphing Relationships

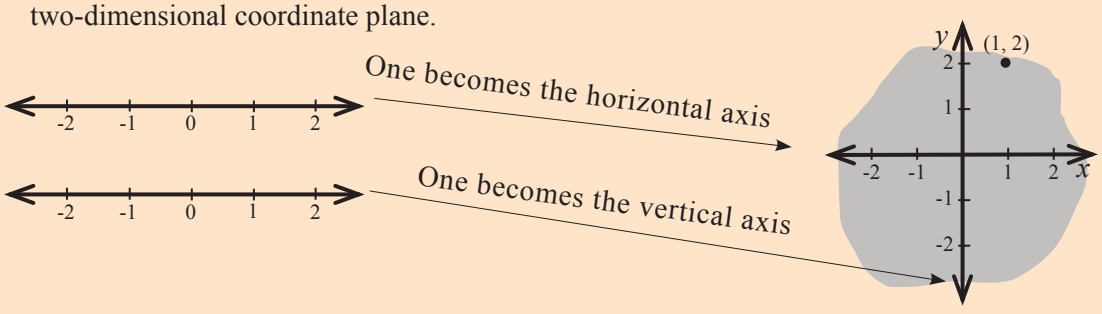
I am (x, y). Where Am I?
I am halfway between (1, 3) and (3, 1).
(,)



You will solve puzzles like these...

Algebraic Habits of Mind: Using Tools

The coordinate plane is really just two number lines put together. Just like a number line, the coordinate plane (or grid) lets you compare locations, measure distances, and tell you where things *are*. It is a tool which allows you to investigate relationships between measurements on two different number lines. Using two number lines means we need two values, or coordinates, to tell where exactly we are on the two-dimensional coordinate plane.



The horizontal axis is often (but not always!) the x-axis, and the vertical is often the y-axis.

 DRAFT

6-1 Plotting Data

Algebraic Habits of Mind: Communicating Clearly

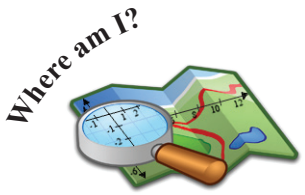
Presenting information simply and compactly makes it easy to communicate. That is why the coordinate plane can be so useful. You can pack a lot of information you draw on a graph. For example, in a graph of armspan vs. height of students in your class, each dot (point) represents a specific person and contains two pieces of information:

1) How tall that person was
2) What that person's armspan was

A whole classroom's height and armspan data can fit on a graph and communicate all the information clearly. Also, because the result is a picture and not just a collection of numbers, you can also see patterns in the data that would otherwise be harder to recognize.

The Where Am I? puzzles are back, in two dimensions!

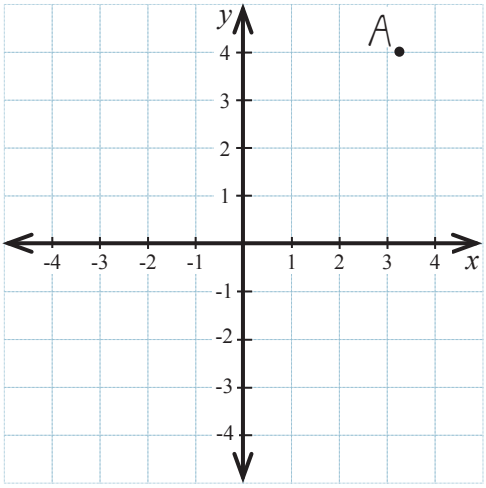
Use the clues to figure out the coordinates of each mystery point. Fill in your answer, and draw and label your solution on the coordinate plane provided.



① I am point A.
My horizontal position is 3.
My vertical position is 4.
(3 ,)

② I am point B.
My x-coordinate is -3.
My y-coordinate is 4.
(,)

③ I am point C.
My x-coordinate is 2.
My y-coordinate has the same value as my x-coordinate. ($y = x$)
(,)



④ I am point D.
My horizontal position is -1.
My vertical is -4.
(,)

⑤ I am point E.
My horizontal position is 2.
My y-coordinate is double my x-coordinate. ($y = 2x$)
(,)

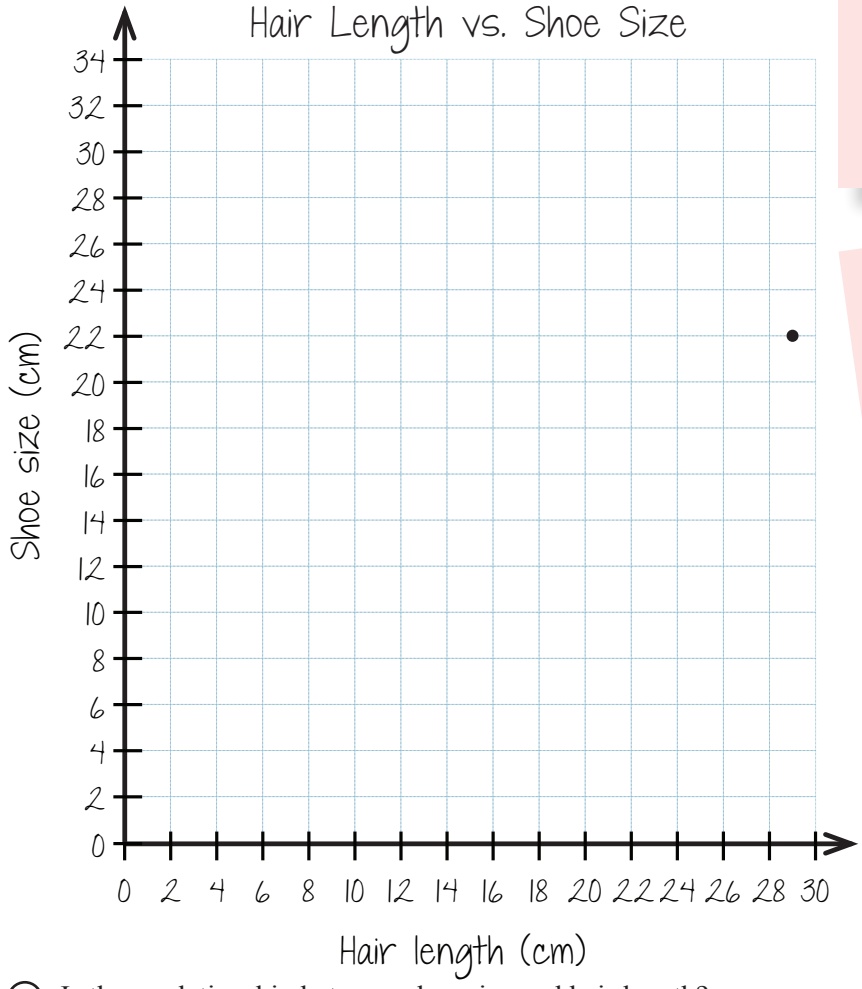
⑥ I am point G.
My x-coordinate is 2.
My y-coordinate is the negative of my x-coordinate. ($y = -x$)
(,)

⑦ I am point F.
My x-coordinate is 3.
My y-coordinate is half the value of my x-coordinate. ($y = \frac{1}{2}x$)
(,)

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

A One student measured her classmates' hair length and shoe size to see if they were related. Plot this information on the coordinate plane provided. Imani's information has already been plotted.



Imani
Hair: 29 cm
Shoe: 22 cm

Eva
Hair: 29 cm
Shoe: 25 cm

Hiroshi
Hair: 17 cm
Shoe: 22.5 cm

Mali
Hair: 29.5 cm
Shoe: 20 cm

Luis
Hair: 27 cm
Shoe: 26 cm

Carla
Hair: 32 cm
Shoe: 24 cm

Jacob
Hair: 21 cm
Shoe: 24 cm

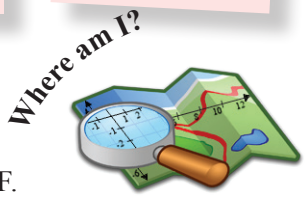
Raj
Hair: 6 cm
Shoe: 26.5 cm

Asher
Hair: 4 cm
Shoe: 27 cm

Brandon
Hair: 5 cm
Shoe: 22 cm

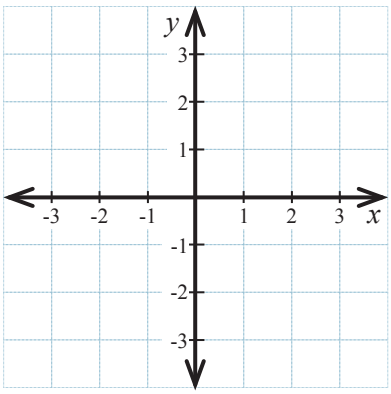
B Is there relationship between shoe size and hair length?

C Why wouldn't you ever see (27, 2) plotted on this graph?



Fill in the coordinates and draw and label each point.

D I am point D.
My horizontal position is -2.
My vertical is 3.
(,)



E I am point E.
My *x*-coordinate is 3.
My *y*-coordinate is -3.
(,)

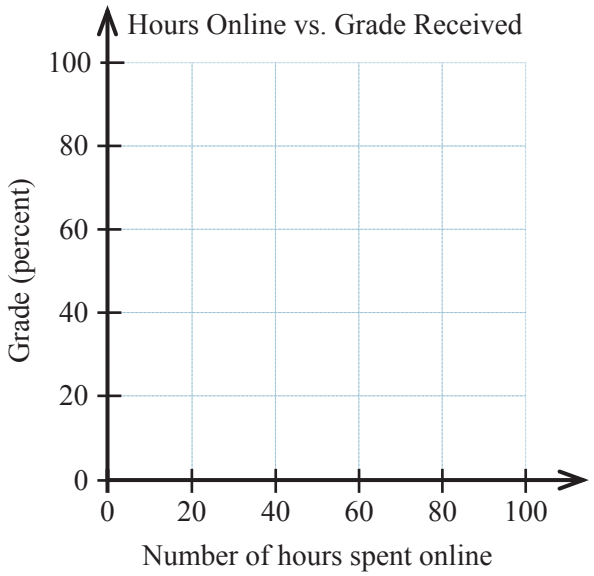
F I am point F.
My horizontal position is -2.
My vertical is -1.
(,)

G I am point G.
My *x*-coordinate is 1.
My *y*-coordinate is double that.
(,)

6-2 Coordinating Data

The table below shows real data from a 'hybrid' algebra class (half the class was online, half in a classroom). The study recorded how many hours students spent studying online during the whole course, and their grade at the end of it.

Student	Number of hours spent online	Grade (%) in algebra class
1	75	82
2	98	66
3	45	71
4	33	60
5	8	9
6	50	88
7	71	91
8	25	33
9	48	80
10	49	95



(1) Plot the data.

(2) In general, does the graph show a relationship between amount of time studying online and grade in the course?

(3) Based on this graph, what amount of time would you spend online if you wanted to score higher than 80% in the course?

Thinking out Loud

Lena: Some of these questions can be answered without drawing the graph. So why do we need it?

Michael: That sounds like something I would ask.

Lena: I guess working with you has changed the way I think.

Michael: Well, I must have changed too, because I think I can answer your question. It's true that the table has the same information as the graph, but it's hard to look at a lot of numbers at once.

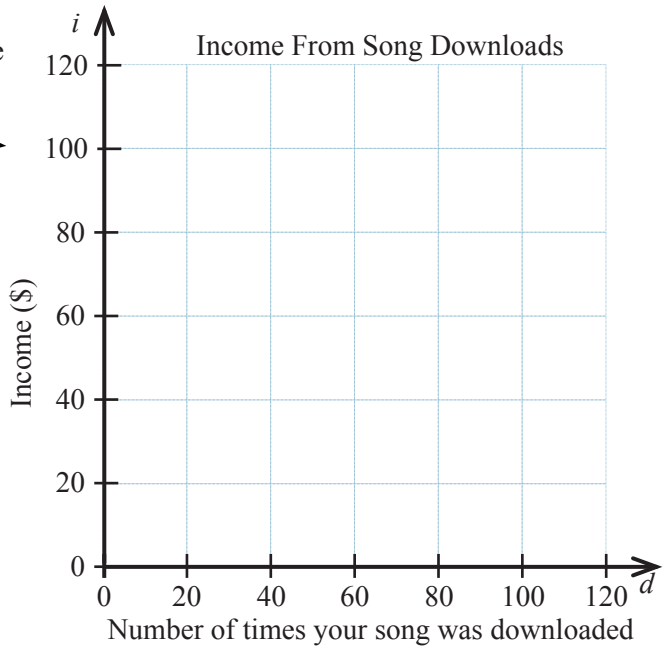
Jay: Yeah. When you turn the numbers into dots on a graph, you can look at a whole bunch of them at the same time. And you can make predictions about points that aren't on the graph. You can't do that as easily with a table. Plus, a graph is a picture. I would choose a picture over a huge stack of numbers any day!

Algebraic Habits of Mind: Seeing and Describing Patterns

In Problems 1–3 you had to figure out how the points on the graph were related. In the next set of problems, you'll make a graph where you already know how the dots are related, you just have have to figure out where they are. Both of these patterns are much easier to visualize using a graph on the coordinate plane.

④ Imagine you made a great song available for sale online at $2 for every download. Use the table to list example scenarios. Then plot them.

Downloads (d)	Income ($) ($i = 2d$)
7	14
10	
22	
35	



⑤ How many downloads would you need to sell to make $120?

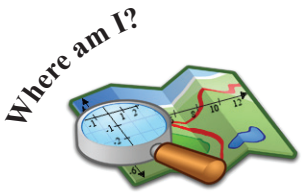
⑥ Would the point (61, 122) be on this graph if the grid were large enough to plot it?

⑦ How about the point (82, 162)?

The vertical value (income) is **always** double the horizontal value (number of downloads). For every *one* download, you earn __ dollars.

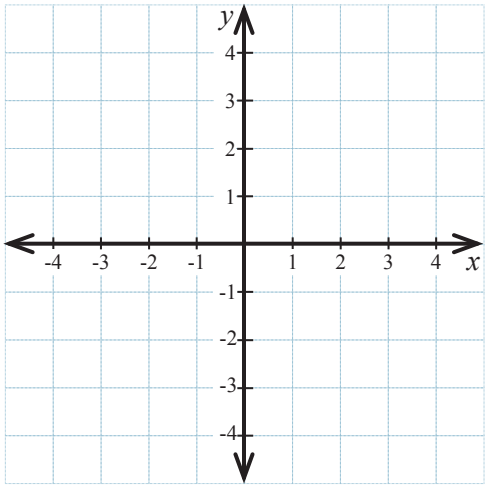
Discuss and Write

Could you make a million dollars from this one song? Explain how you know.



Fill in the coordinates, and draw label each point.

⑨ I am point R.
My horizontal position is 3.
My vertical position is one third my horizontal position. ($y = \frac{1}{3}x$)
(,)



⑩ I am point T.
I have the same y-coordinate as (-1, -2) and the same x- coordinate as (0, 8).
(,)

⑫ I am point D.
I have the same x-coordinate as (3, -2) and the same y- coordinate as (-1, 4).
(,)

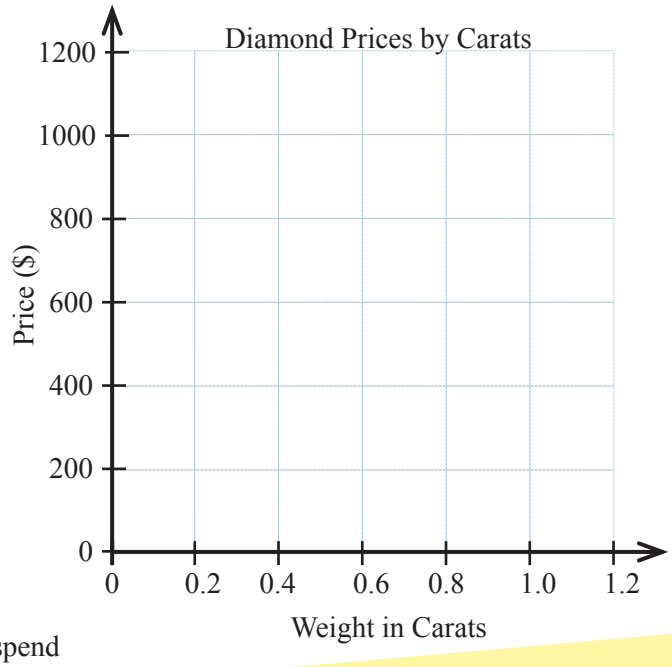
⑬ I am point Q.
I have the same y-coordinate as (0, 1) and the same x- coordinate as (-4, 1).
(,)

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

A The table below shows the prices and weights of six diamonds from a single retailer as of October, 2011. A one-carat diamond weighs about the same as an ordinary small paper-clip. We can graphs the data to make help us analyze it.

Diamond	Weight (Carats)	Price ($)
1	0.1	203
2	0.5	449
3	0.2	300
4	0.85	801
5	0.25	520
6	1.0	1044



B How much does diamond #3 cost?

C Which diamond weighs the most?

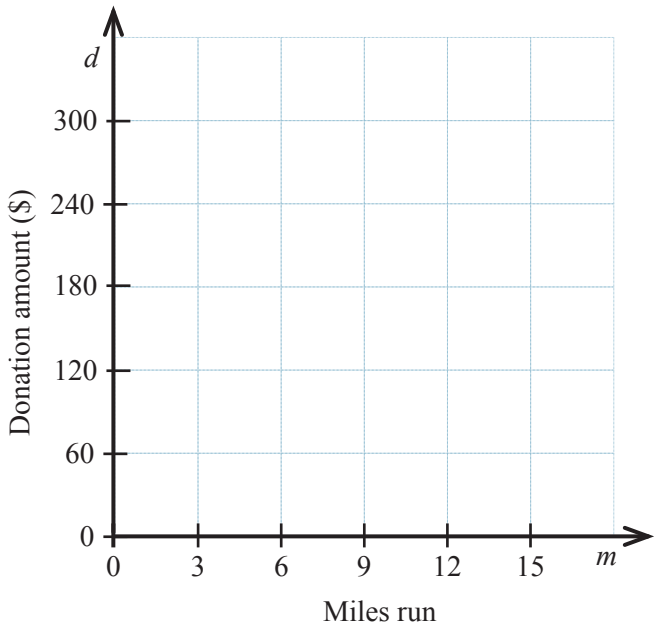
D Approximately how much would you have to spend on a 0.6 carat diamond?

E If you go just by cost and weight, which diamond is the best deal?

Each point represents one particular diamond, and contains two pieces of information about it: its weight, and its price

F Imagine you decided to do a long distance race to benefit health research. Your school agrees to donate $20 for every mile you run. Fill in the table with some examples and plot this information on the graph.

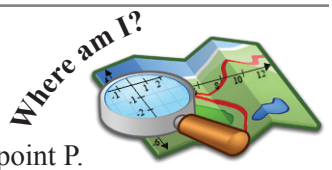
Miles (m)	Donation ($d = 20m$)
1	20
3	



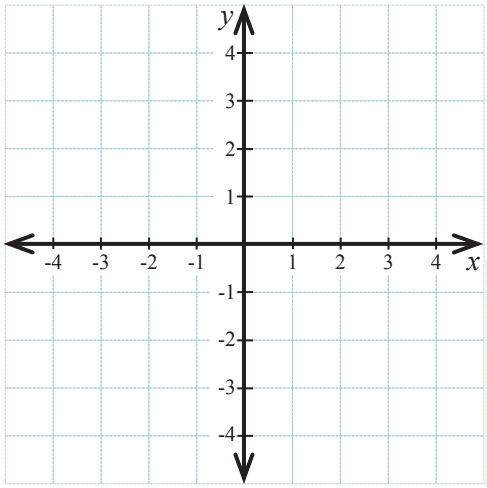
G If you wanted to make **more than** $270, how far would you have to run?

Additional Practice Problems

Fill in the coordinates, and plot and label each point.



(H) I am point U.
I have the same y-coordinate as (3, -3) and the same x-coordinate as (2, 4).
(,)

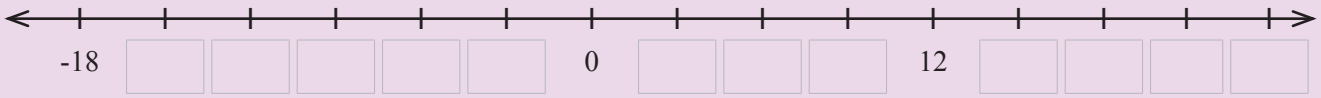


(J) I am point P.
I am half way between (0, 0) and (4, 4).
(,)

(I) I am point L.
My horizontal position is -1.
My vertical position is double my horizontal position. ($y = 2x$)
(,)

(K) I am point M.
I am half way between (-2, 0) and (-2, 4).
(,)

(L) Finish labeling this number line.



Just like you can represent different scales on a one-dimensional number line, you can use different scales along the axes of a coordinate plane.

(M) What's the distance between -20 and -81? _____

(N) What about between -34 and 80? _____

(O) How far apart are -45 and -13? _____

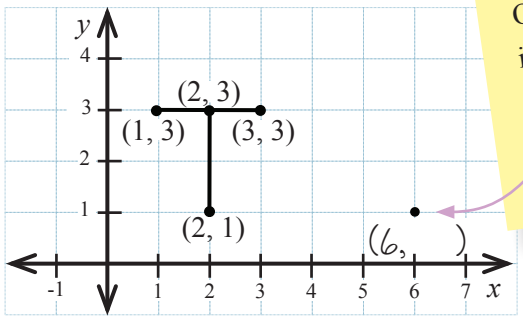
(P) How far apart are 91 and 413? _____

(Q) What is the distance between -6 and 504? _____

(R) What is the distance between 3 and -6.1? _____

6-3 Geometric Transformations

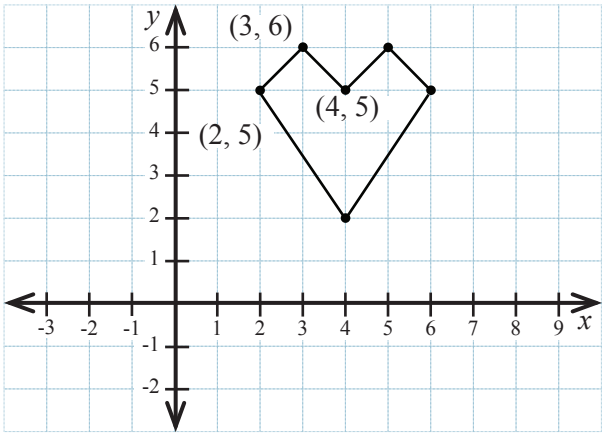
① **Move** this diagram **4 units** to the **right** and **draw** it again. **Record** all of the points in the table, and **label** them on the graph.



One point is already marked. What is the y-coordinate?

points on original shape	image points after moving 4 to the right
(2, 1)	(6 ,)
(2, 3)	(, 3)
(3, 3)	
(1, 3)	
(x, y)	$(x + 5, y)$

② Use the transformation rule $(x, y) \rightarrow (x, y - 4)$ to redraw this diagram **4 units down**. **Record** all of the points in the table and **label** them on the graph.

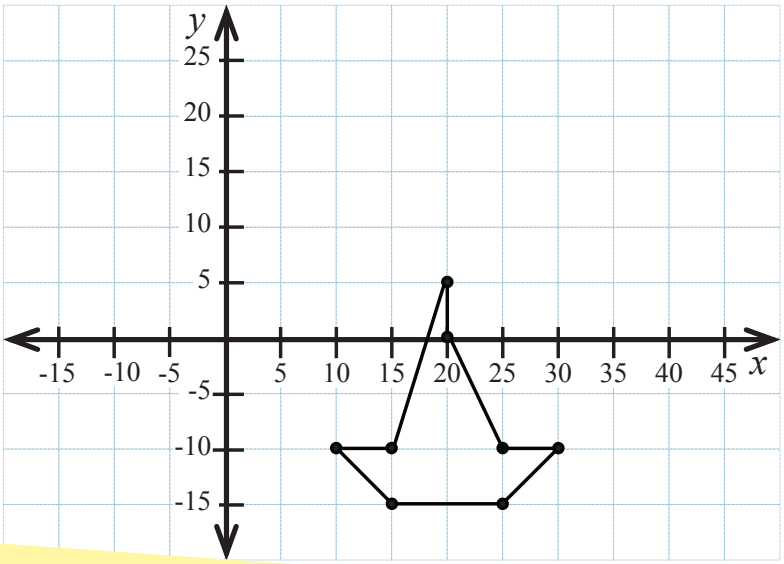


points on original shape	image points after moving 4 units down
(3, 6)	
(4, 5)	
(2, 5)	
(x, y)	$(x, y -)$

Discuss and Write

In Problem 2, where would the shape move if the transformation had been $(x, y) \rightarrow (x - 4, y)$ instead? Write how you know. If you want, graph it as well.

③ Use the transformation rule **$(x, y) \rightarrow (x + 15, y + 10)$** to move and **redraw** this graph. **Record** all of the points in the table and **label** them on the graph.



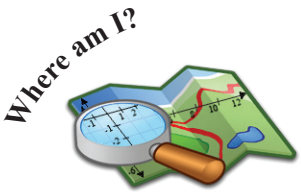
points on original shape	points after transformation
(10, -10)	
(15, -15)	
(15, -10)	
(30, -10)	
(x, y)	$(x + 15, y +$ $)$

Make sure to look at the scale! This grid uses 5-unit increments.

This notation shows what happens to **all** of the points on the graph, not just the corners.

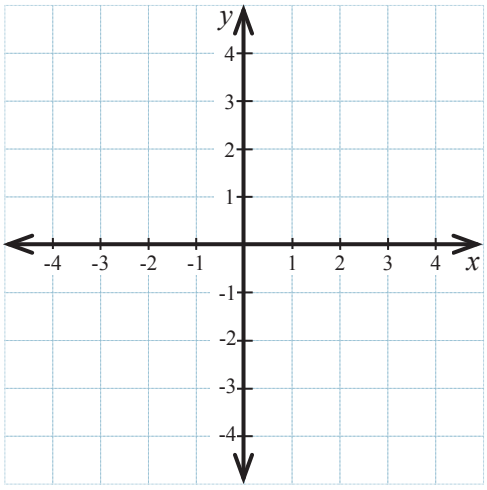
Fill in the coordinates, and plot and label each point.

④ I am point K.
My horizontal position is -1.5.
My vertical position is double my horizontal position. ($y = 2x$)
(,)



⑦ I am point J.
I am half way between (0, 0) and (-4, 4).
(,)

⑤ I am point M.
My y-coordinate is 3.
$y = x$
(,)



⑧ I am point N.
My x-coordinate is -1.
$y = 3x$
(,)

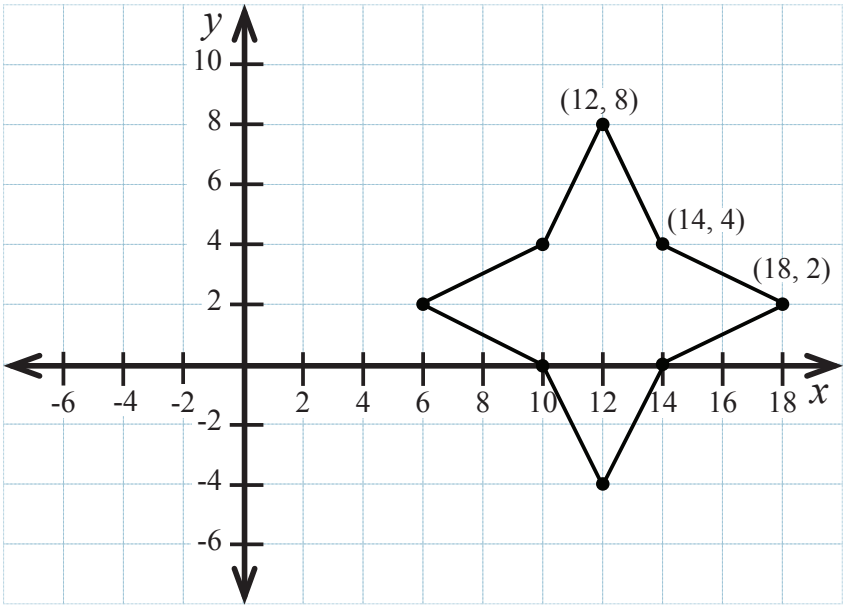
⑥ I am point R.
My horizontal position is -4.
My vertical position is half my horizontal position. ($y = \frac{1}{2}x$)
(,)

⑨ I am point P.
I am half way between (0, 0) and (0, -4).
(,)

Additional Practice Problems

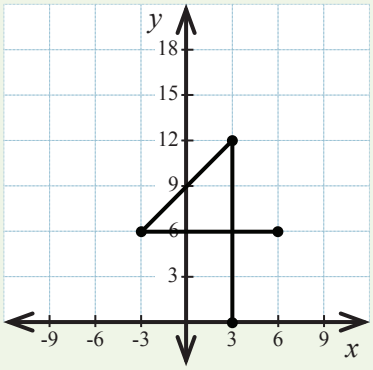
Select problems that will help you learn. Do some problems now. Do some later.

Ⓐ Use the transformation rule **(x, y) → (x – 12, y)** to move and **redraw** this graph. **Record** all of the points in the table.



points on original shape	points after transformation
(12, 8)	
(14, 4)	
(18, 2)	
(x, y)	(x - , y)

Ⓑ Experiment with this graph and table to find out what **(x, y) → (x, y + 3)** does. Note the scale on both axes.

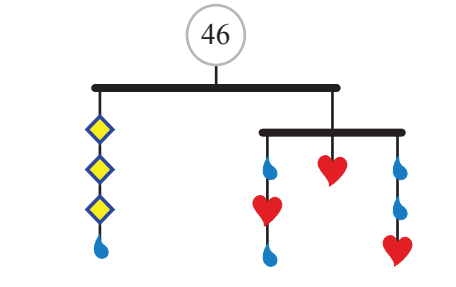


(x, y)	(x, y + 3)
(6, 6)	(6, 9)

Ⓒ If the transformation in problem B had been **(x, y) → (x + 3, y)** describe or draw what the graph would look like.

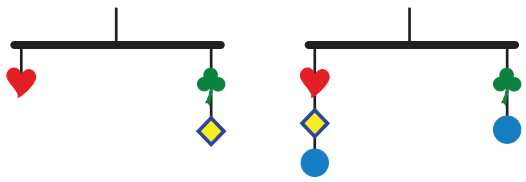
Ⓓ What if the transformation had been **(x, y) → (x + 3, y + 3)**?

Ⓔ



♥ = _____ ◆ = 7 💧 = _____

Ⓕ

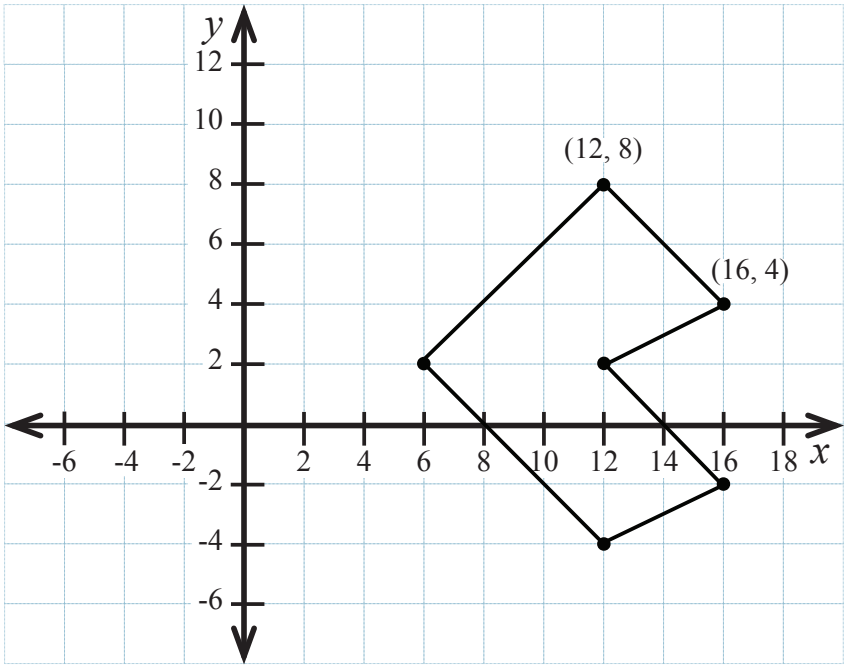


If this mobile balances... does this? YES or NO

Explain:

Additional Practice Problems

G Use the transformation rule $(x, y) \rightarrow (x - 6, y - 2)$ to move and redraw this graph. Record all of the points in the table.

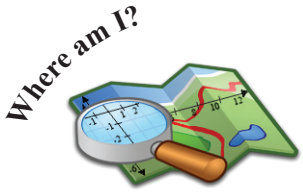


points on original shape	points after transformation
(12, 8)	
(16, 4)	
(x, y)	(x - , y -)

H What if the previous transformation had been $(x, y) \rightarrow (x + 2, y + 6)$? Describe or draw what the graph would look like.

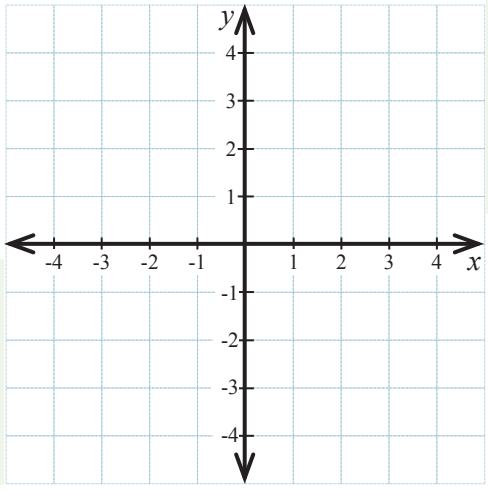
Find the coordinates. Plot and label each point.

I I am point I.
I have the same y-coordinate as (-1, 2) and the same x-coordinate as (0, 4).
(,)



L I am point L.
I am half way between (-4, 4) and (-4, -4).
(,)

J I am point J.
My horizontal position is -2
My vertical position is double my horizontal position. ($y = 2x$)



M I am point M.
I am half way between (0, -3) and (4, 1).
(,)

K I am point K.
My y-coordinate is 4.
($y = x$)
(,)

N I am point N.
My x-coordinate is 1.
($y = 3x$)
(,)

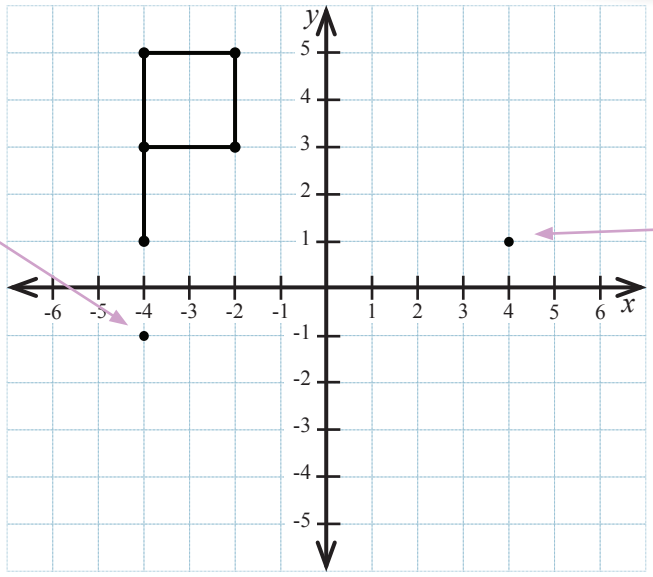
6-4 Transformations With Algebra

These two transformations do very different things to a graph, but each is called a "reflection." Ask yourself why...

① Apply these **two different** transformations to the graph below. Fill in the table and draw the new shape.

Transformation
(x, y) → (x, -y)

(x, y)	(x, -y)
(-4, 1)	(-4, -1)

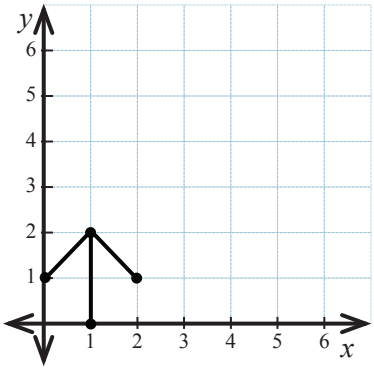


Transformation
(x, y) → (-x, y)

(x, y)	(-x, y)
(-4, 1)	(4, 1)

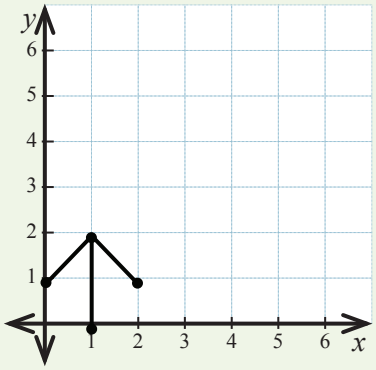
For the next three questions, use the different transformation rules to fill out the tables and redraw the arrow in three different ways.

② **(x, y) → (2x, 2y)**



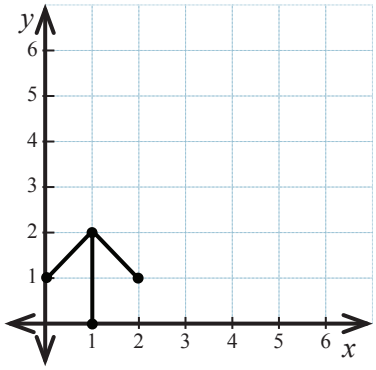
(x, y)	(2x, 2y)
(0, 1)	

③ **(x, y) → (3x, 3y)**



(x, y)	(3x, 3y)
(0, 1)	

④ **(x, y) → (x, 3y)**



(x, y)	(x, 3y)
(0, 1)	

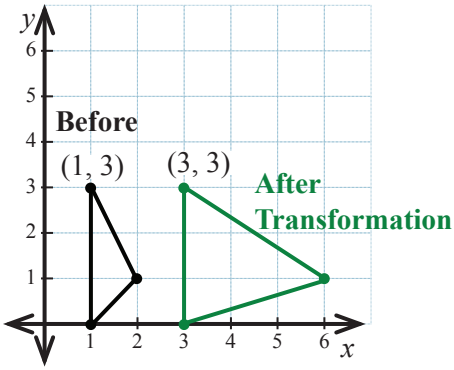
⑤ Which transformation made the arrow the largest? Why?

Look at the x and y separately here. What happened to all the x-values? Now look at all the y-values.

Below is a graph of a triangle, **before** and **after** it has been transformed in three different ways. Fill out the tables and use the graphs to discover how each triangle was transformed.

⑥ Transformation

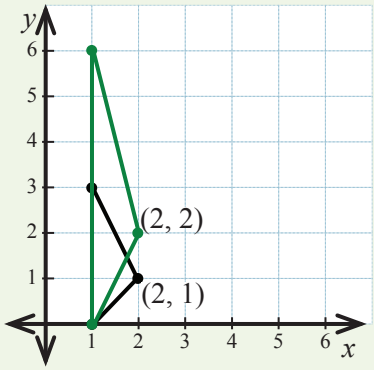
$(x, y) \rightarrow (\quad , \quad)$



(x, y)	$(\quad , \quad)$
(1, 3)	(3, 3)

⑦ Transformation

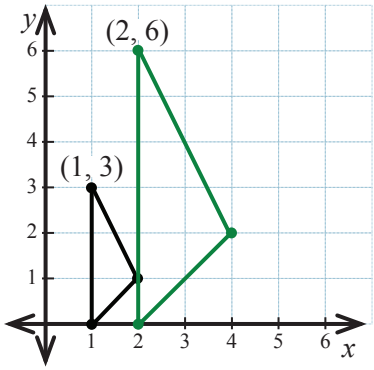
$(x, y) \rightarrow (\quad , \quad)$



(x, y)	$(\quad , \quad)$
(2, 1)	(2, 2)

⑧ Transformation

$(x, y) \rightarrow (\quad , \quad)$



(x, y)	$(\quad , \quad)$
(1, 3)	(2, 6)

Thinking out Loud

Michael, Lena, and Jay are working on Problems 6-8.

Michael: There are actually a lot more than three points in these triangles.

Jay: What? A triangle has three points.

Michael: A triangle has three *corners*, Jay. It also has three sides. In these images, we transformed the corner points and just connected them with lines, but what about the points that make up the sides? Like (1, 1.273)?

Lena: I never really thought of it that way, but you're right. Every point on the graph is transformed at the same time so we could choose any one of them.

Jay: Um, except we don't know the location of every point on the triangle, do we?

Pausing to Think What does Jay mean by "we don't know the location of every point on the triangle"?

Michael: Ugh, Jay's right. Let's stick to the corners!

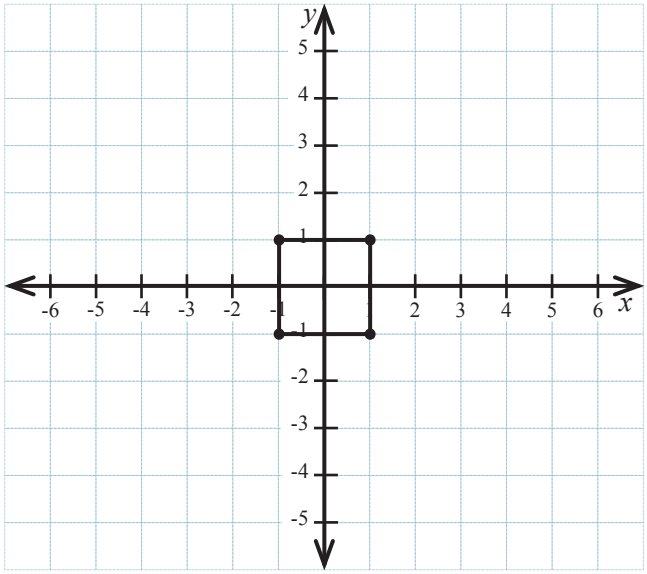
Challenge: If the transformation rule was $(x, y) \rightarrow (x^2, y)$? Would Jay's method of transforming corners and connecting them work?

⑧ Make this box **taller**, but **not wider** using your own transformation rule.

My transformation

$(x, y) \rightarrow (\quad , \quad)$

(x, y)	$(\quad , \quad)$
$(1, 1)$	



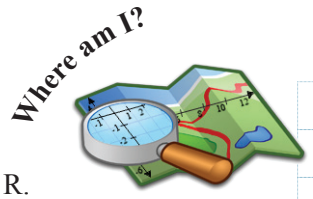
⑨ What did you do to the x-coordinates?

⑩ What did you do to the y-coordinates?

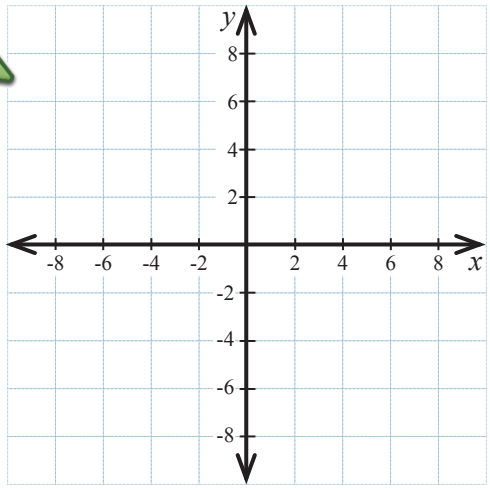
⑪ Explain how this transformation made the box taller but not wider.

Discuss and Write

How might you make the box **wider**, but ***not taller***?



⑫ I am point R.
My x-coordinate is -6.
$y = x$
(, -6)



⑬ I am point T.
$y = 2x$
My x-coordinate is -3.
(,)

⑭ I am point A.
My x-coordinate is 4.
$y = x$
(,)

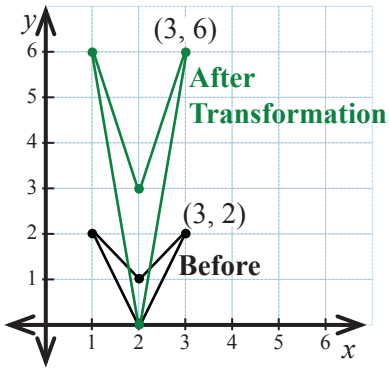
⑮ I am point C.
My x-coordinate is -2.
$y = 4x$
(,)

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

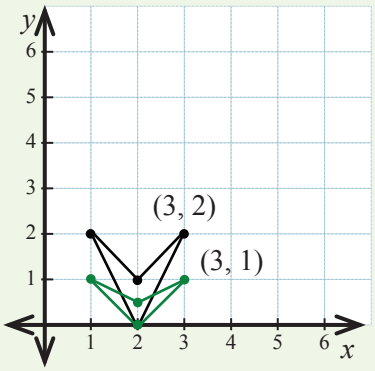
Below is a graph, **before** and **after** it has been transformed in three different ways. Fill out the tables and use the graphs to discover how each graph was transformed.

Ⓐ $(x, y) \rightarrow (\ x\ ,\ \ \ )$



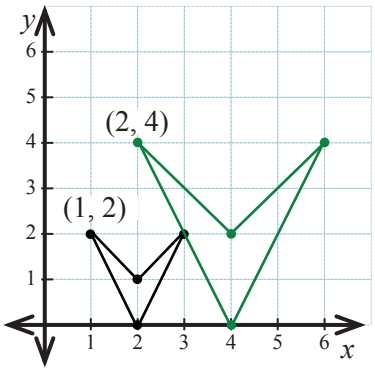
(x, y)	$(\ x,\ \ \ )$
(3, 2)	(, 6)

Ⓑ $(x, y) \rightarrow (\ \ \ ,\ \ \ )$



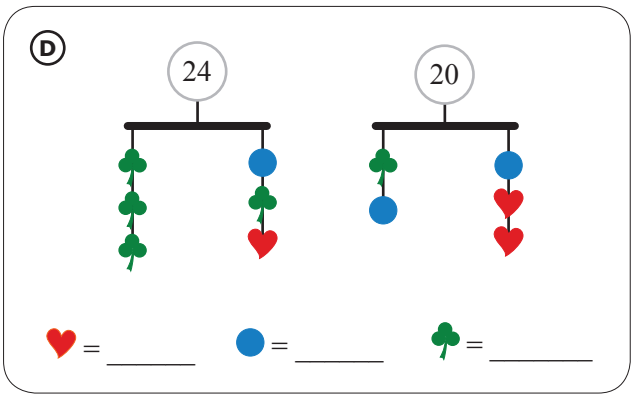
(x, y)	$(\ \ ,\ \ )$
(3, 2)	(3,)

Ⓒ $(x, y) \rightarrow (\ \ \ ,\ \ \ )$



(x, y)	$(\ \ ,\ \ )$
(1, 2)	(, 4)

Ⓓ



Ⓔ MysteryGrid **1-3-5-7** Puzzle

5	9, x	35, x	
21, x			
	17, +		1
		21, x	

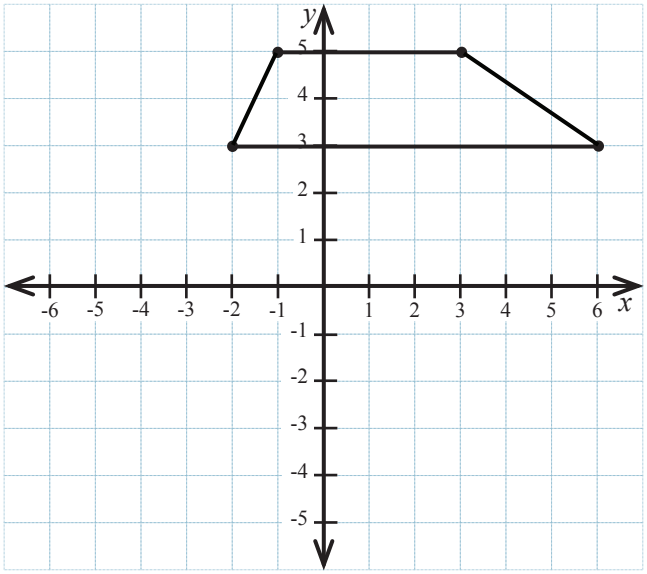
Additional Practice Problems

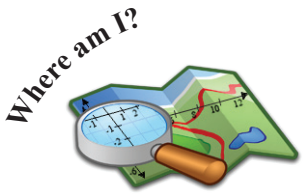
F Make this shape smaller in some way using your own transformation. Fill out the transformation and the table.

My transformation

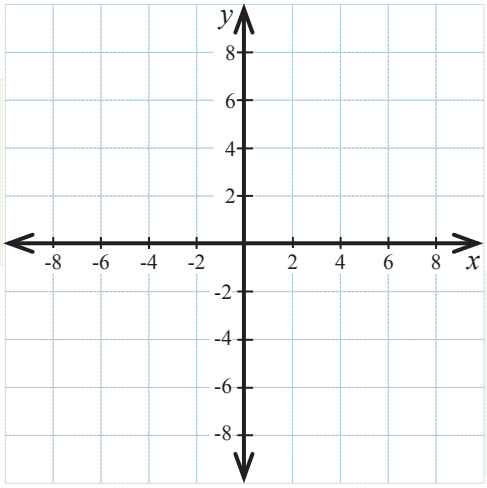
$(x, y) \rightarrow (\quad , \quad)$

(x, y)	$(\quad , \quad)$
$(-2, 3)$	





Find the coordinates. Plot and label each point.



G I am point R.
My x-coordinate is 6.
$y = x$
(, 6)

H I am point T.
$y = 2x$
My x-coordinate is -4.
(,)

I I am point A.
My x-coordinate is -7.
$y = x$
(,)

J I am point C.
My x-coordinate is 2.
$y = 4x$
(,)

Solve.

K $(c - 2)^2 = 64$

That's [redacted]$^2 = 64$.
So what's $c - 2$? c?

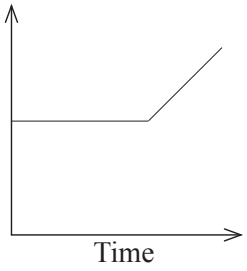
L $(r + 5)^2 = 49$

M $(4 + n)^2 = 81$

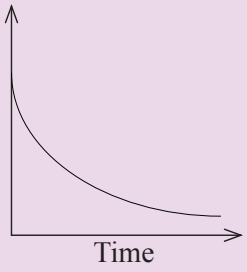
N $(4x + 2)^2 = 100$

6-5 Intuitive Graphing

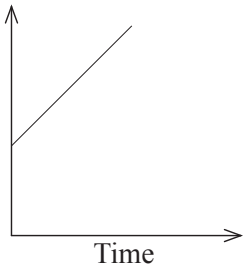
Cut out the matching scenarios from the handout and glue them here.

① 

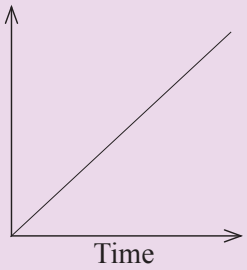
Attach matching scenario(s) here.

② 

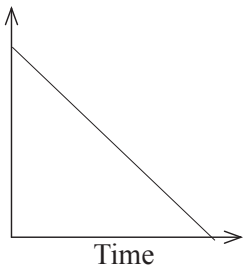
Attach matching scenario(s) here.

③ 

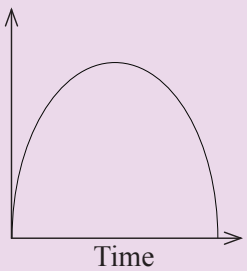
Attach matching scenario(s) here.

④ 

Attach matching scenario(s) here.

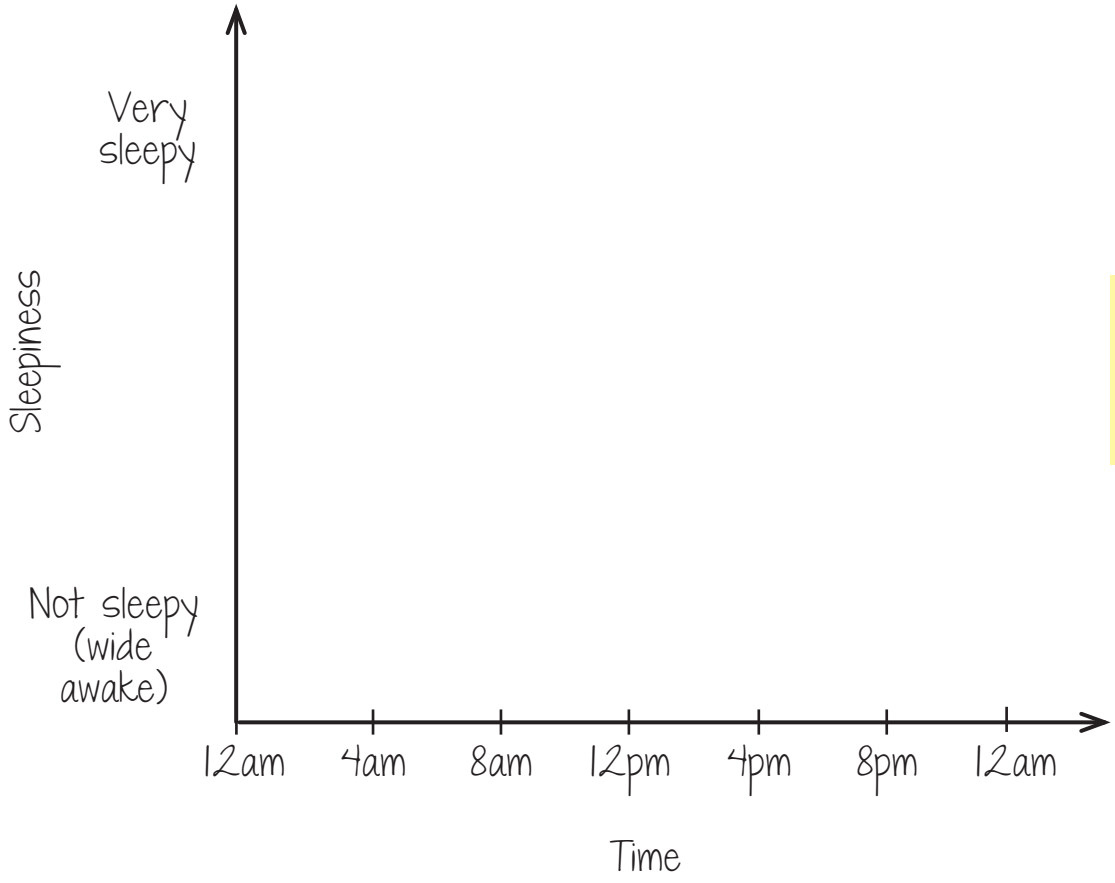
⑤ 

Attach matching scenario(s) here.

⑥ 

Attach matching scenario(s) here.

⑦ We have looked at graphs of many different relationships. But now we're going to look more closely... at ourselves. Make a graph of how sleepy you feel over the course of the 24 hour day. Have fun with this and try to make it accurate!



Compare with a classmate. Talk about the differences.

⑧ What time of day are you usually the sleepiest? How does the graph show that?

⑨ Describe the graph during the times you are asleep.

Thinking out Loud

Michael and Lena are working on Question 7.

Michael: Wait. The vertical axis doesn't have numbers, just words. "Very sleepy" is not a number.

Lena: Maybe it's not a number, but it's an amount. We don't know what "very sleepy" means, but we know it's more than "not very sleepy," right?

Michael: (Yawns) Right!

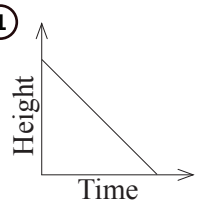
Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

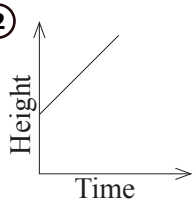
A Choose the best graph to fit the following scenario:

"When they moved in, the apple tree was only one foot tall, but it grew more each year."

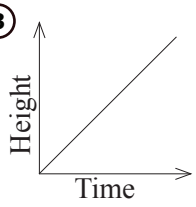
1



2



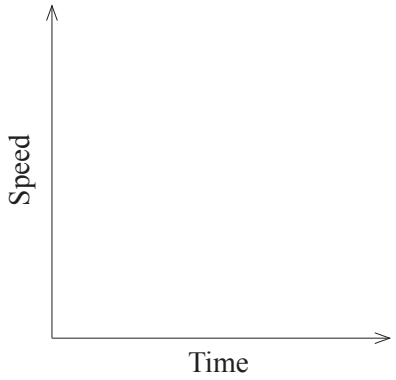
3



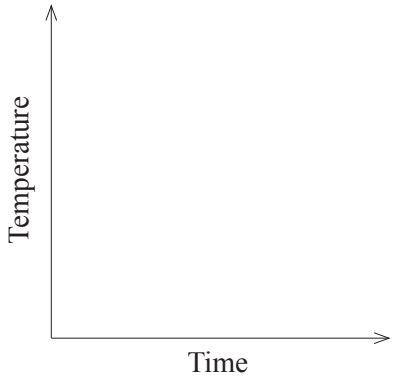
B Which axis is for the number of shrubs and which is for time? Label each axis on the graph you picked.

Sketch your own graph to illustrate these scenarios:

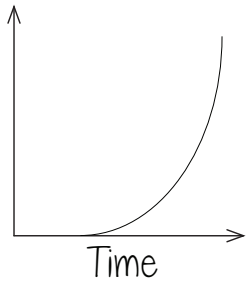
C "The train was going very fast and had to slow down to go around a turn, but it sped right back up again."



D "Once the sun went down, the temperature started dropping. In the morning, when the sun rose again, it warmed right back up.."



E Choose the best scenario to match this graph:



1 "Malika filled the pitcher of water steadily until it could hold no more."

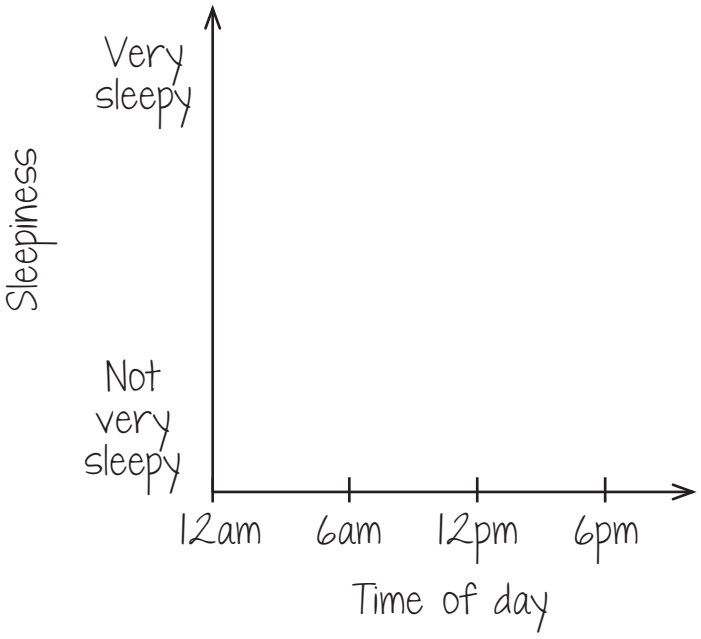
2 "Ben was always short, but suddenly he started growing taller until he was 6 feet tall."

3 "The email virus started in just a few accounts, but it was emailed to all of their contacts, and then mailed to all of *their* contacts, until thousands of accounts were infected."

F Label the vertical axis in the previous problem to match the scenario you picked.

Additional Practice Problems

Ⓖ During the summer, Brandon works the overnight shift, from 11pm - 7am, then he goes home to sleep. Draw what you think his sleep graph might look like.



Ⓗ Compare Brandon's sleep graph with your own.

Ⓘ Brandon gets the weekends off. What might his sleep graph look like on those days?

Ⓙ MysteryGrid **1-2-3-4** Puzzle

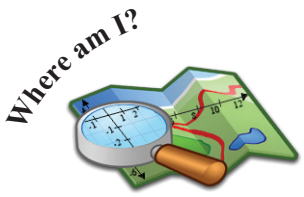
8, +		9, x	
	4	4, x	
7, +			16, x

Ⓚ MysteryGrid **2-3-5-7** Puzzle

18, x		98, x	5
	50, x		
22, +			1, –

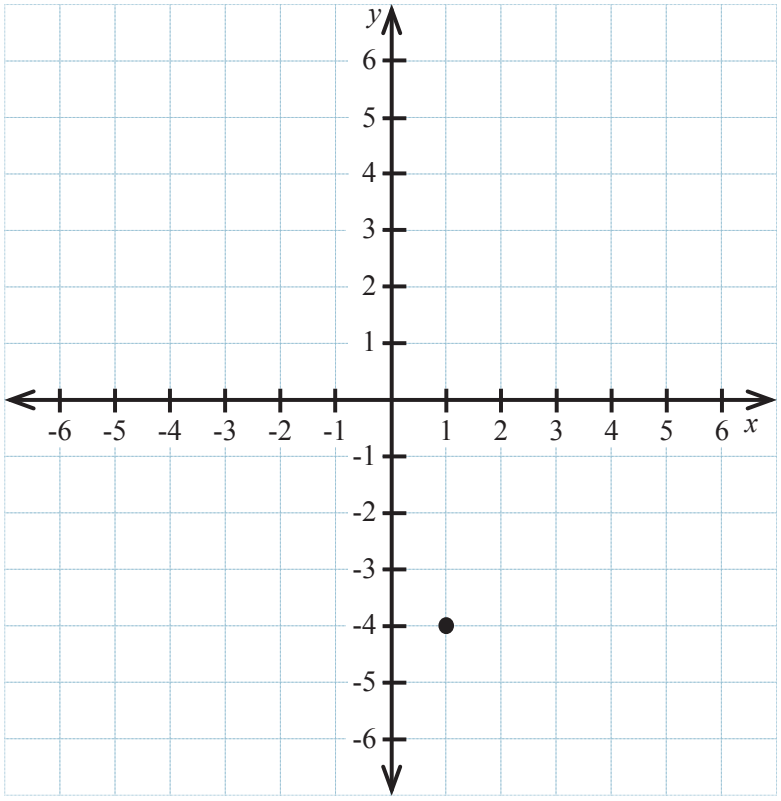
6-6 Where Could I Be? Solutions and Point Testing

I am point A. Where could I be?
Clue: $y = x + -5$
Hint: there are many, many solutions.



① Circle the points that *are* solutions, and plot them on the grid if possible. Cross out all of the non-solution points.

(45, 89) (1, -4) $(\frac{1}{2}, -4\frac{1}{2})$ (-6, 1) (2, -3)
(3, -2) (0, -5) (10, 5) (46, 90) (5, 0)



② Find six more solution points for this equation. You may not be able to plot all of them.

$y = x + -5$

x	y
(	,)
(	,)
(	,)
(	,)
(	,)
(	,)

The y-coordinate **always** equals $x + -5$!

③ Connecting your solution points will suggest other solution points, in between the ones you found. List two that look like solutions, and check to see if they are.

Algebraic Habits of Mind: Seeing and Describing Patterns

If we plotted enough solution points, the result would look like a line. This is why equations like $y = x + -5$ are plotted as a line. The graph shows infinitely many solution points on the coordinate plane.

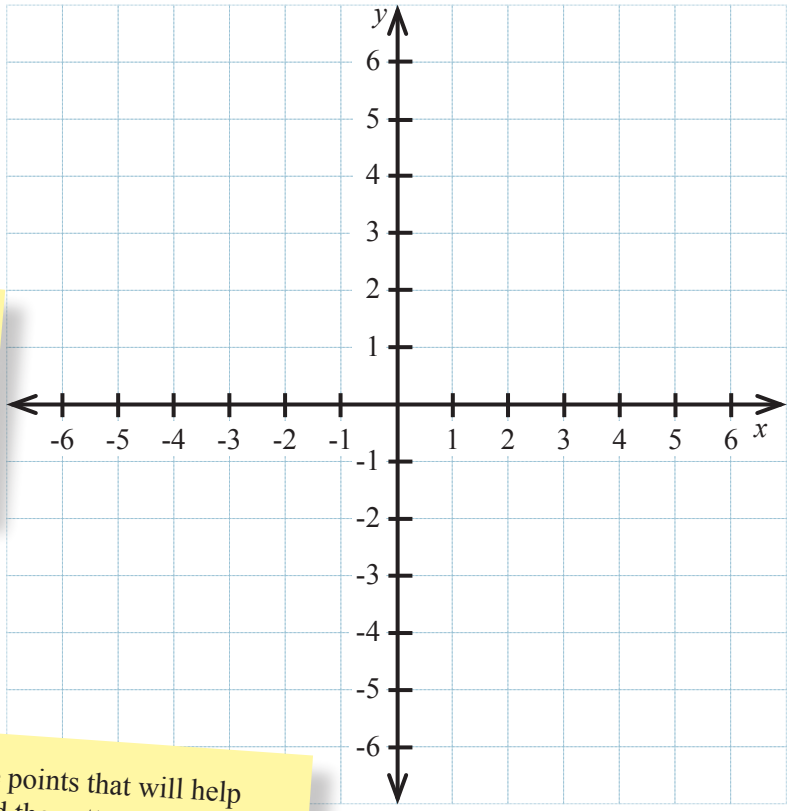
Circle the solution points and plot them if possible. Plot six of your own solution points and connect them.

④ $y = 2x - 1$

(2, 3) (-3, -7) (-1, -3)
(-25, -51) (30, 53) (1, -1)

x	y
(,)	
(,)	
(,)	
(,)	
(,)	
(,)	

The y-coordinate **always** equals $2x - 1$.

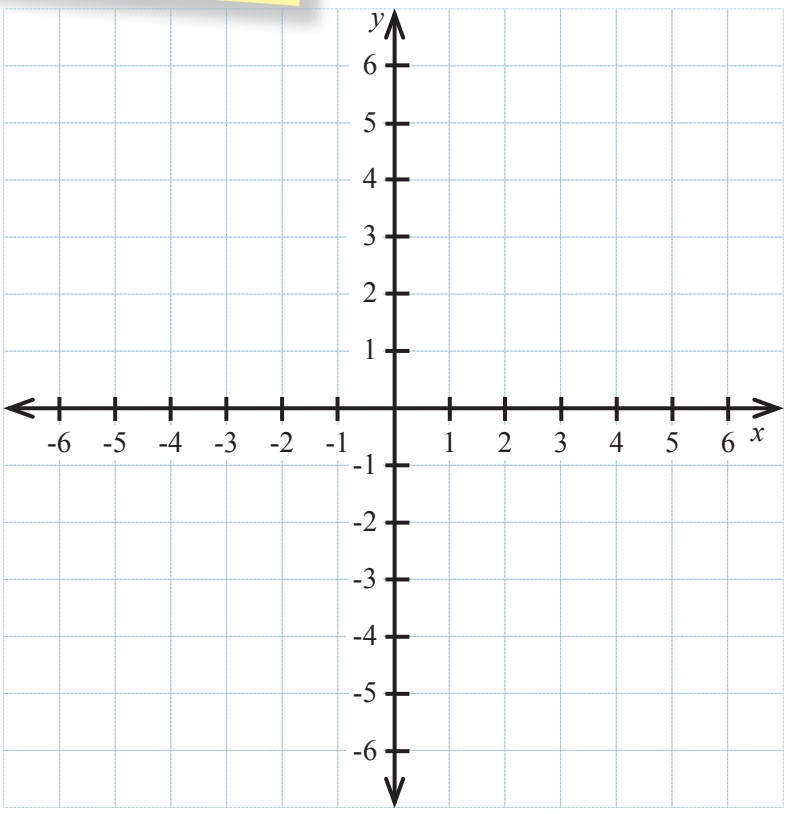


Choose points that will help you find the pattern. Use some negative numbers.

⑤ $y = x^2 - 5$

(-3, -7) (10, 95) (1, -1)
(-2, -1) (2, -1) (-9, 76)

x	y
(,)	
(,)	
(,)	
(,)	
(,)	
(,)	



Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

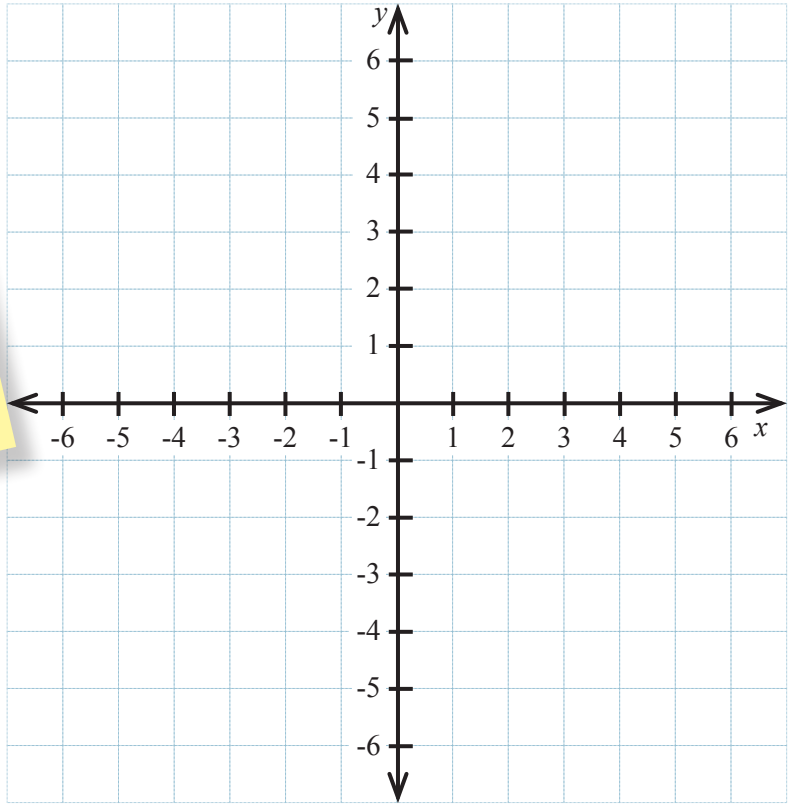
Circle the solution points and plot them if possible. Plot six of your own solution points and connect them.

Ⓐ $y = -x + 1$

$(2, 3)$ $(40, -39)$ $(-5, 6)$ $(-3, -2)$ $(2, -1)$ $(-1, 2)$

x	y
(,)	
(,)	
(,)	
(,)	
(,)	
(,)	

The y-coordinate **always** equals $-x + 1$.

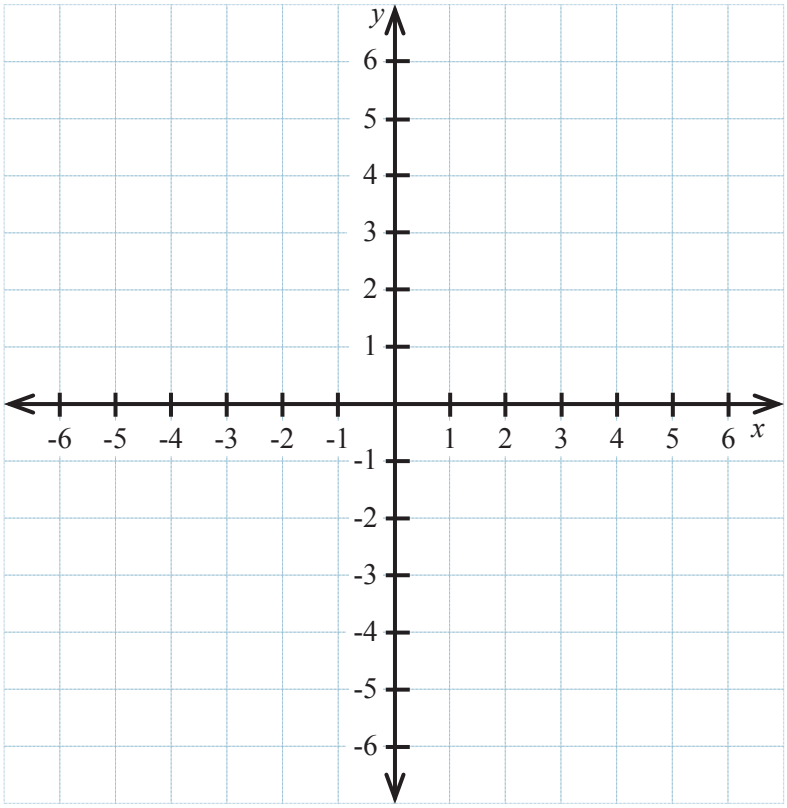


Square the x-value first, then take half of it.

Ⓑ $y = \frac{1}{2}x^2$

$(-2, 2)$ $(1, \frac{1}{2})$ $(2, 2)$ $(8, 32)$ $(-7, 51)$ $(-4, 6)$

x	y
(,)	
(,)	
(,)	
(,)	
(,)	
(,)	



6-7 Graphing Relationships

Fill in the solution points for each equation, then cut and paste the matching graph from the handout.

① $y = x + 4$

Record solution points in the table.

x	-4	0	1		
y					

Attach the matching graph here.

② $y = 2x - 1$

Record solution points in the table.

x	0	1	4		
y					

Attach the matching graph here.

③ $y = 4$

Record solution points in the table.

x	-4	0	2		
y					

Attach the matching graph here.

④ $y = -3x - 2$

Record solution points in the table.

x	-2	0			
y					

Attach the matching graph here.

⑤ $x = -3$

This is **not** $y = -3$

Record solution points in the table.

x					
y	-3	0	3		

Attach the matching graph here.

⑥ $y = x^2 - 3$

Record solution points in the table.

x	-2	0	2		
y					

Attach the matching graph here.

⑦ $y = -\frac{1}{2}x + 4$

Record solution points in the table.

x	-2	0	4		
y					

Attach the matching graph here.

⑧ $y = -x^2 + 5$

Record solution points in the table.

x	-1	0	2		
y					

Select a value, square it, find the opposite of it, then add 5.

Attach the matching graph here.

Additional Practice Problems

Select problems that will help you learn. Do some problems now. Do some later.

Find 5 solution points for each equation and use them to match the equations to the graphs. Use easy numbers for x.

Ⓐ $y = -2x - 1$

x	-2	0			
y					

Graph: _____

Ⓑ $y = 3x + 2$

x	-2	0			
y					

Graph: _____

Ⓒ $y = \frac{1}{2}x - 1$

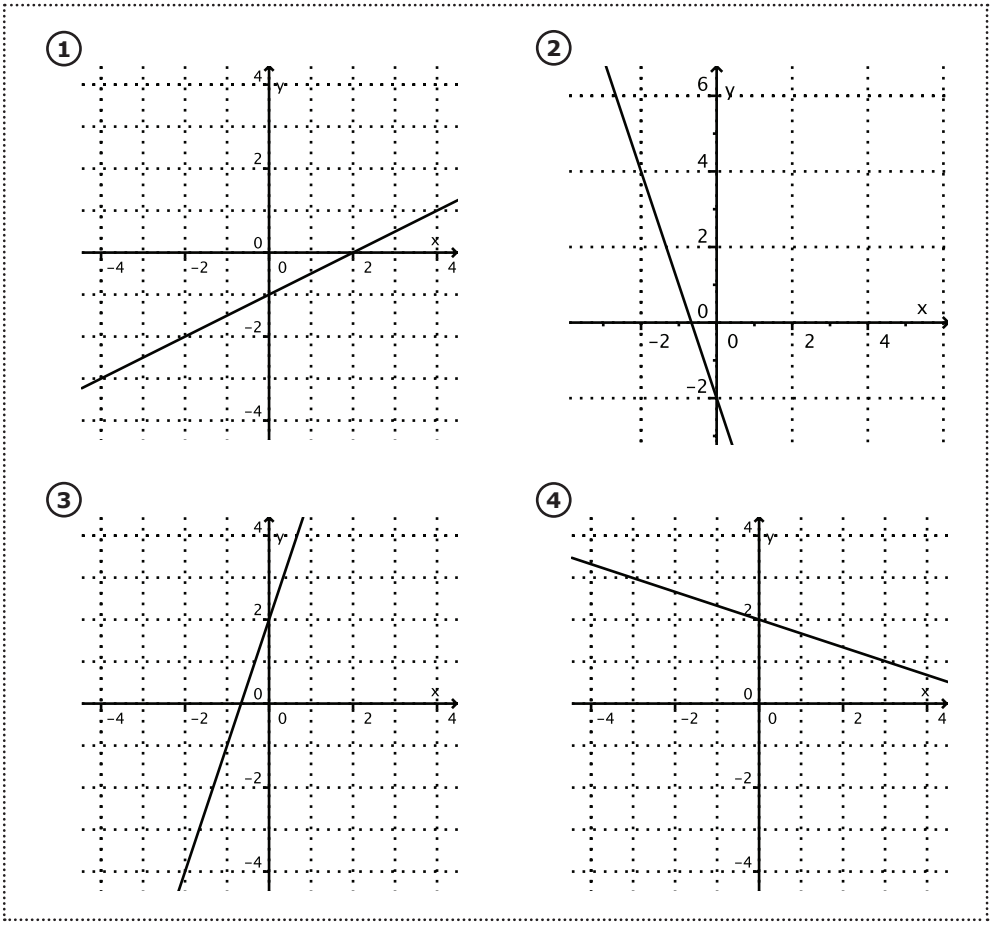
x	2	1	0		
y					

Graph: _____

Ⓓ $y = -\frac{1}{3}x + 2$

x	-3	0			
y					

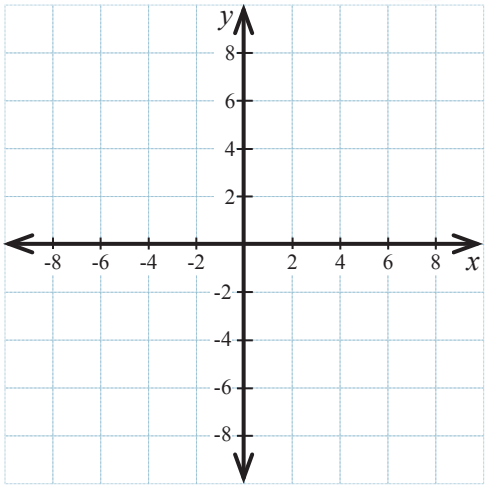
Graph: _____



Unit Additional Practice Problems

Use these pages to prepare for the unit exam.

Find the coordinates. Plot and label each point.



① I am point A.
My x-coordinate is -4.
My y-coordinate is 8.
(,)

② I am point B.
$y = -x$
My x-coordinate is 6.
(,)

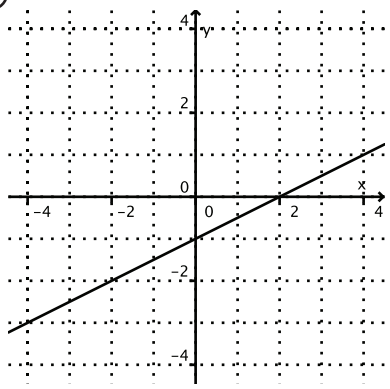
Find 5 solution points for this equation and select the matching graph.

③ $y = \frac{1}{2}x - 1$

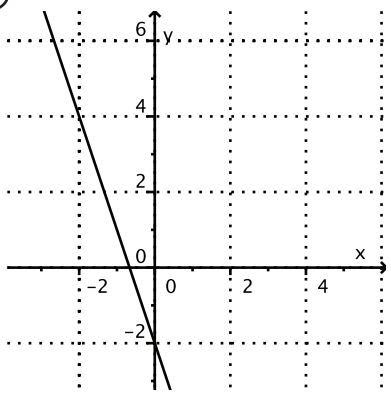
x	2	0			
y					

Graph: _____

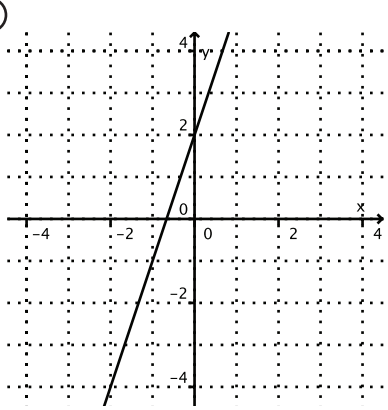
Ⓐ



Ⓑ



Ⓒ

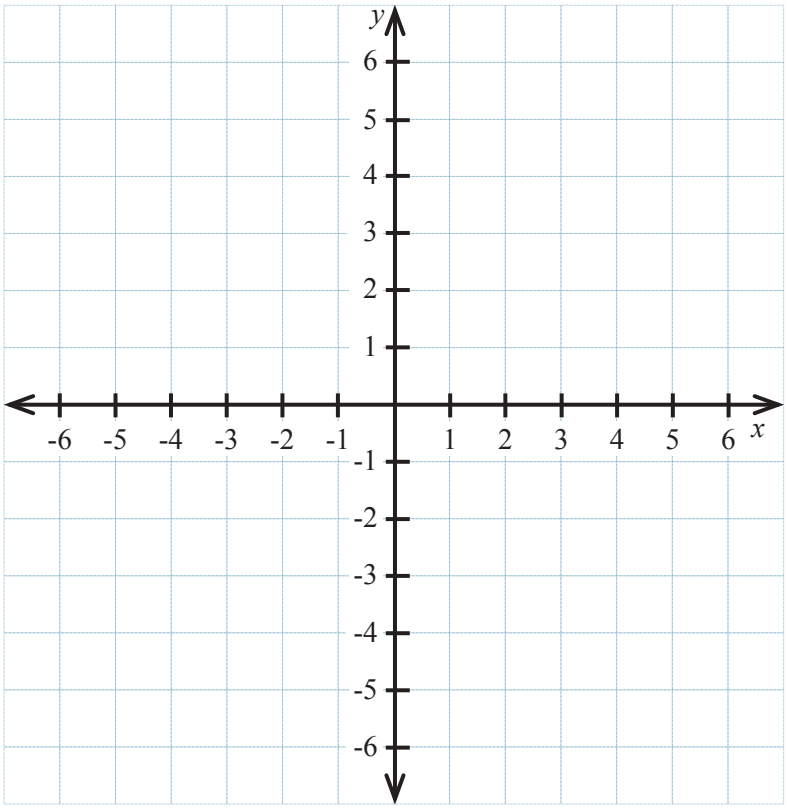


Circle the solution points and **plot** them if possible. Plot three of **your own solution points** and **connect them all**.

④ $y = -2x + 2$

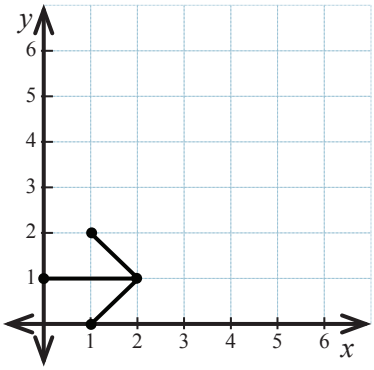
(0, 2) (1, 0) (-2, 6)
(-2, 2) (10, -18) (-20, -38)

x	y
(	,)
(	,)
(	,)



Use the transformation rule to fill out the tables and redraw the arrow.

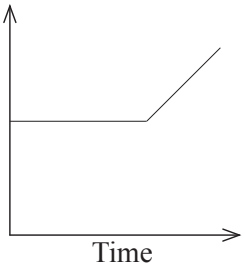
⑤ **$(x, y) \rightarrow (3x, 2y)$**



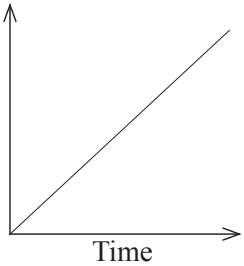
(x, y)	$(3x, 2y)$
(0, 1)	

⑥ The puppy didn't weigh much when they brought her home, and she gained weight steadily every day. Which graph shows her weight over time since they got her?

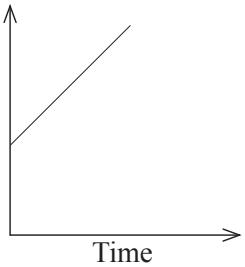
Ⓐ



Ⓑ

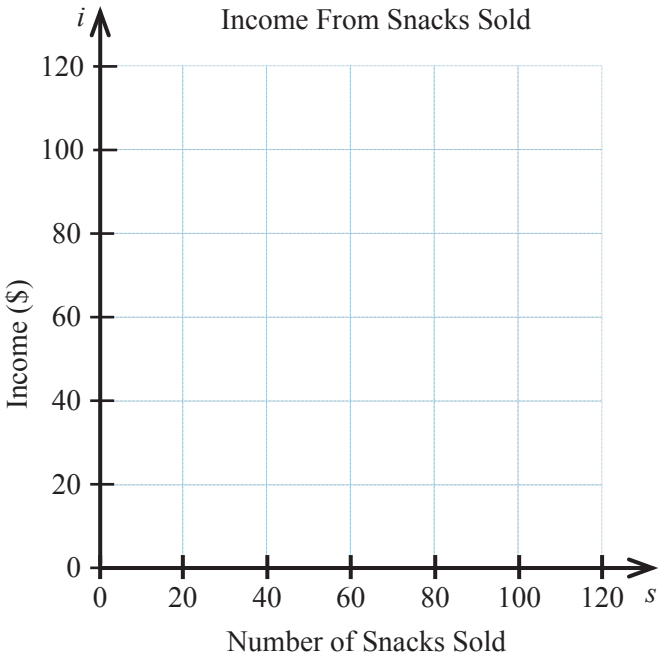


Ⓒ



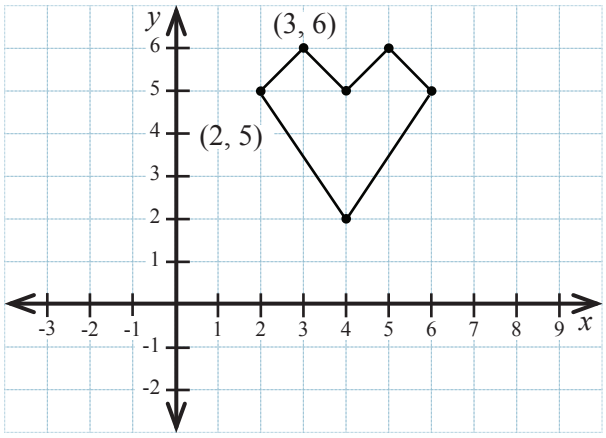
⑦ Imagine you sold snacks for 50¢ each. Fill out the table and the graph to show the total fundraiser income for different numbers of snacks sold.

Snacks (s)	Income ($) ($i = \frac{1}{2}s$)
20	10
40	
50	



⑧ How many snacks would you need to sell to make $80?

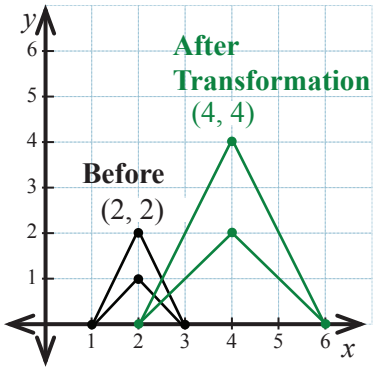
⑨ Use the transformation rule **$(x, y) \rightarrow (x, y - 3)$** to redraw this diagram **3 units down**.
Record all of the points in the table and **label** them on the graph.



points on original shape	image points after moving 3 units down
(3, 6)	
(2, 5)	
(x, y)	$(x, y - 3)$

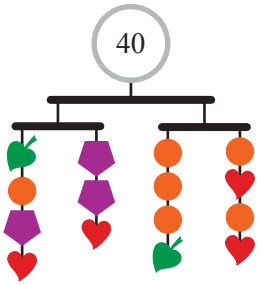
Below is a graph, **before** and **after** it has been transformed.
Fill out the table and use the graph to discover how the graph was transformed.

⑩ **$(x, y) \rightarrow$ (,)**



(x, y)	(,)
(2, 2)	(,)

Transition to Algebra



Transition to Algebra is an EDC project supported by the National Science Foundation aimed at *very quickly* giving students the mathematical knowledge, skill, and confidence to succeed in a standard first year algebra class.

The familiar topic-oriented approach to mathematics is replaced by a small number of key mathematical ideas and ways of thinking: Algebraic Habits of Mind (puzzling, using structure, generalizing patterns, using tools, and communicating clearly). Conventional algebra topics are part of the curriculum, but instead of the topics being the focus of the lesson, they become contexts for exploring these broadly-applicable problem-solving strategies.

EDC Learning transforms lives.

More information on this research and development project is available at ttalgebra.edc.org.